East Side Alliance
Mathematics Toolkit

2021-2022 Edition

The East Side Alliance *includes the East Side Union High School District and the seven elementary partner districts: Alum Rock, Berryessa, Evergreen, Franklin-McKinley, Mt. Pleasant, Oak Grove, and Orchard.*

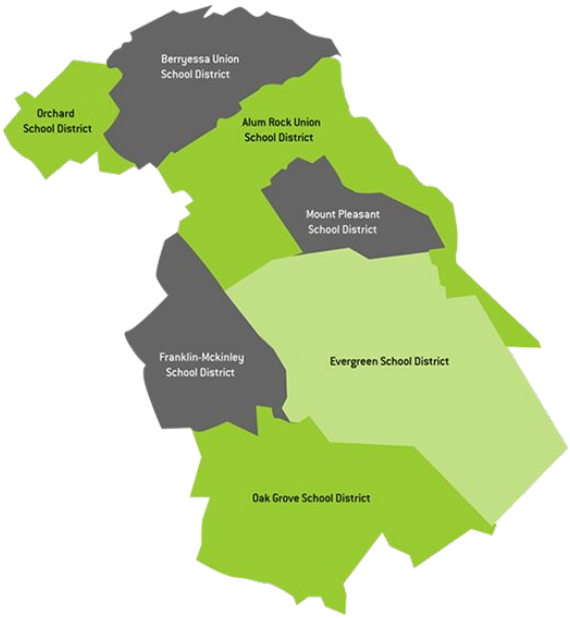
Table of Contents

East Side Alliance

The East Side Alliance Math Toolkit

Why this Toolkit?

The East Side Alliance (ESA) consists of eight partner districts: the East Side Union High School District and the seven elementary partner districts: Alum Rock, Berryessa, Evergreen, Franklin-McKinley, Mt. Pleasant, Oak Grove, and Orchard. Together they serve approximately over 70,000 students—almost the combined size of San Francisco and Oakland Unified School Districts. If the ESA were one district, it would be the third largest district in the state.



Over the past several years the East Side Alliance (ESA) has focused on implementing the mathematics Common Core State Standards (CCSS). The CCSS transforms what should be taught to students with a new emphasis on going deeper into mathematical concepts than perhaps might have been expected in the past. The CCSS has brought on a significant change in expected instructional practices to properly ensure that students in fact learn the content standards themselves. The CCSS implementation has generated changes in curriculum, instruction and assessment. Students are no longer only expected to know algorithms and formulas but also to think deeply and make sense of the mathematical practices they are learning. The new Smarter Balanced Assessments require students to explain their answers to reveal a deeper understanding of the mathematics involved. Helping students express their mathematical thinking has become an essential component of what students need to learn.

Learning mathematics has become increasingly important. Research shows that success in high school mathematics is a strong indicator of future success in college (Adelman). Success in math as early as third grade is a strong indicator of future success in school (Gingsburg). With this in mind, mastering the rigorous CCSS has become increasingly challenging. The purpose of this Toolkit is to provide a common focus on what instructional practices may be most helpful in helping students master grade level CCSS.

The ESA Math Toolkit was developed with a team comprised of both internal and external math experts, along with input from the most important expert of all: the classroom teacher. It is designed to be organic and potentially changing as we learn together what math instructional best practices look like. It is not intended to be exhaustive in nature but rather serve as a guide for districts and schools across the Alliance to help focus their instruction in the most effective way.

Acknowledgments

Lisa Andrew Ed.D - CEO, Silicon Valley Education Foundation

Dr. Lisa Andrew is the CEO of the Silicon Valley Education Foundation, the largest educational non-profit in the Bay Area. Prior to coming to SVEF, she was the Superintendent of the Hollister School District. Lisa's nonprofit service started in 2014 when she served as the Regional Executive Director of Partners in School Innovation. Before her work at Partners, Lisa worked as a Director at the Santa Clara County Office of Education, working on system reform and assessment and accountability development. Lisa has held several positions in education, including teacher, coach, principal, Director, Assistant Superintendent, and college professor. Her reputation as a strong advocate for all students has led to invitations to present on school reform, leadership, curriculum and instruction, English language development, strategic planning, and navigating the political waters of education. Lisa's dedication to the field has been recognized by the Silicon Valley Chamber of Commerce as a Woman in Leadership and is the ACSA 2013 Valuing Diversity state award winner and 2015 Partner in Educational Excellence award winner for Santa Clara County.

Denise Williams – CPO, Silicon Valley Education Foundation

Denise Williams, M.Ed. serves as CPO for the SVEF and is currently coordinating the East Side Alliance. Previously, she served as a teacher, principal and Director of Instruction and Categorical Programs in Evergreen School District. She attended the Equity Academy of Association of California School Administrators(ACSA) and is a member of ACSA. She was the recipient of NAACP Dollarhyde Award for her work in the educational community and also led her previous district in receiving the State Golden Bell Award and the County Kristy Porter Award for Family Engagement. All of this work provides structures for students, teachers and parents to focus not only Mathematics, but building the knowledge base for all students to be prepared for 21st Century Learning.

Ma Bernadette Salgarino, Ed.D., NBCT – Mathematics Coordinator, Santa Clara County Office of Education

Ma Bernadette Salgarino, Ed.D. is a Mathematics Coordinator for the Santa Clara County Office of Education (SCCOE). Bernadette has taught AP Calculus AB/BC, and AP Statistics among others, a mentor and an instructional coach at East Side Union High School District (ESUHSD). She is a National Board Certified Teacher (NBCT), a member of the CA Mathematics Curriculum Framework Committee, an AP Calculus Reader, and MDTP and SBAC item writer and reviewer. She's also a Board Member for CA Mathematics Council (CMC) and TODOS Mathematics For All. Her work on Mathematics for Social Justice led her to co-author TODOS' position statement on "The Mo(ve)ment to Prioritize Antiracist Mathematics." Her classes had been videotaped for NBCT featuring instructional practices that support all learners to develop and strengthen literacy skills and mathematics understanding. She was awarded the Texas Instruments – STEM Teacher of the Year in 2012 and the ESUHSD Teacher of the Year in 2013.

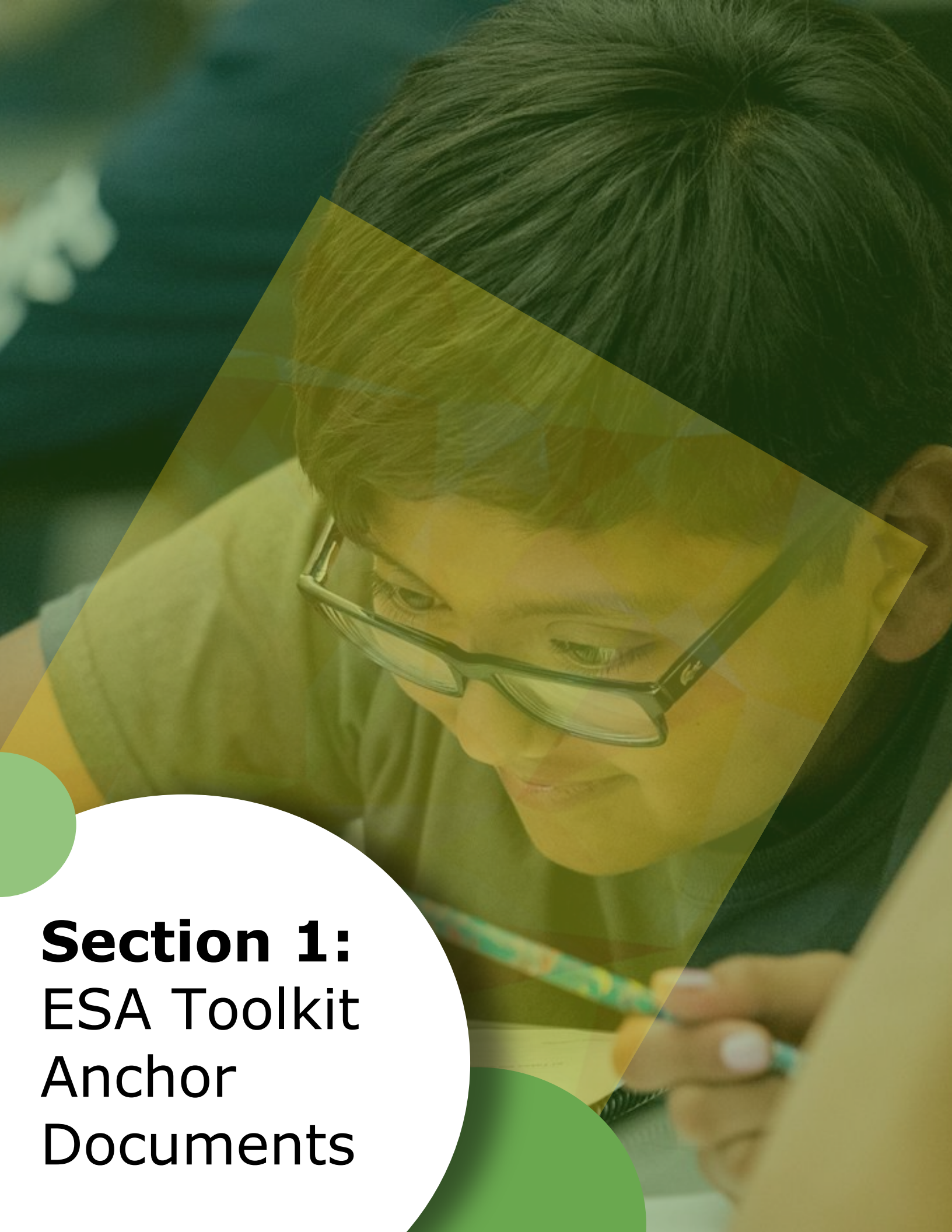
The ESA Math Toolkit was created in collaboration across multiple school districts, as well as with educational partners, including: **Silicon Valley Education Foundation, East Side Alliance & Santa Clara County Office of Education.**

Thank you our advisors:

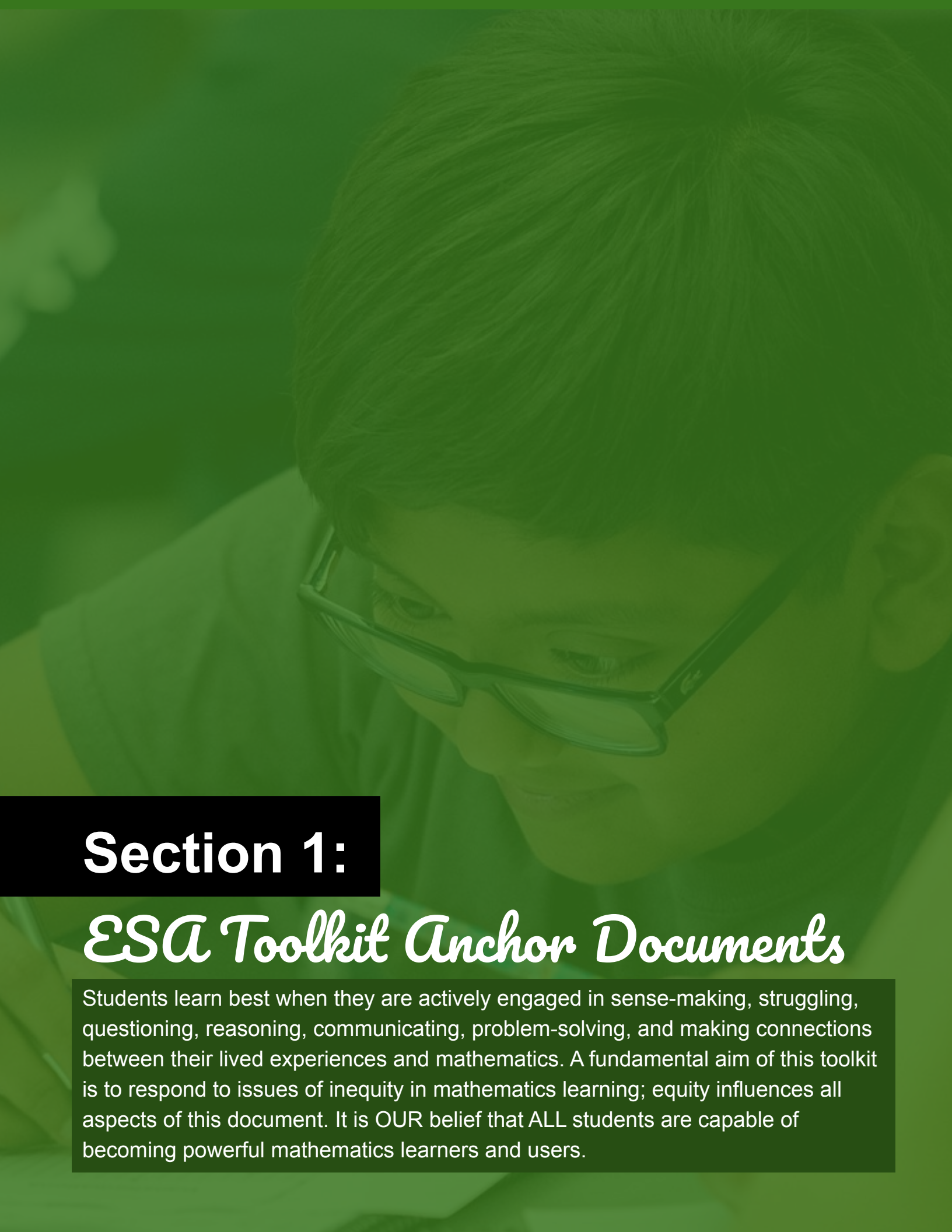
Dan Moser, SVEF
Jivan Dhaliwal, San Benito County Office of Education
Manny Barbara, SVEF

Special thanks to our revision committee members:

Sandhya Raman, BUSD
Gil Agoylo, ESUHSD
Danielle Letts, ARUSD
Sukanya Ramachandran, FMSD
Olena McLin, OGSD
Barbara Powell, MPESD
Diana Stahlnecker, EESD
Flor Perez, OSD
Vy Nguyen, SVEF
Christina Atkinson, SVEF
Gucci Trinidad, SVEF



Section 1: ESA Toolkit Anchor Documents



Section 1:

ESA Toolkit Anchor Documents

Students learn best when they are actively engaged in sense-making, struggling, questioning, reasoning, communicating, problem-solving, and making connections between their lived experiences and mathematics. A fundamental aim of this toolkit is to respond to issues of inequity in mathematics learning; equity influences all aspects of this document. It is OUR belief that ALL students are capable of becoming powerful mathematics learners and users.

Guiding Principles for Mathematics Programs in California

Adapted from Mathematics Framework for California Public Schools

Five guiding principles underlie the Standards for Mathematical Practice, Standards for Mathematical Content, and other resources in this framework; *see table IN-1 below.* These philosophical statements should guide the construction and evaluation of mathematics programs in schools and the broader community. The Standards for Mathematical Practice are interwoven throughout the guiding principles.

Guiding Principle 1: Learning

All students will explore mathematical ideas in ways that will stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

Guiding Principle 2: Teaching

All students will experience a mathematics program that is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence of learning.

Guiding Principle 3: Technology

All students will use technology as an essential tool that will be used strategically in mathematics learning.

Guiding Principle 4: Equity

All students will have a high-quality mathematics program that prepares them for college and careers.

Guiding Principle 5: Assessment

All students will experience many forms of assessment which will inform instruction and monitor student learning.

The above guiding principles are the foundation for the Standards for Mathematical Practice, Standards for Mathematical Content, and a variety of other resources in the *California Mathematics Framework* (www.cde.ca.gov). These principles have been slightly modified so that the focus of mathematics teaching remains clearly on the students. The National Council for the Teachers of Mathematics, *Effective Mathematics Teaching Practices,* strongly supports the California framework. In addition, the Teaching for Robust Understanding (TRU) Mathematics Framework supports the description for powerful classrooms.

Standards for Mathematical Practice

From Mathematics Framework for California Public Schools

The Standards for Mathematical Practice (SMP) describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1) Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2) Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3) Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

4) Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5) Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6) Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7) Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7.

8) Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

A Pathway to Equitable Mathematics Instruction

https://equitablemath.org/

A Pathway to Equitable Math Instruction is an integrated approach to mathematics that centers Black, Latinx, and Multilingual students in grades 6-8, addresses barriers to math equity, and aligns instruction to grade-level priority standards. The Pathway offers guidance and resources for educators to use now as they plan their curriculum, while also offering opportunities for ongoing self-reflection as they seek to develop an anti-racist math practice. The toolkit "strides" serve as multiple on-ramps for educators as they navigate the individual and collective journey from equity to anti-racism.

STRIDE 1: Dismantling Racism in Mathematics Instruction

This tool provides teachers an opportunity to examine their actions, beliefs, and values around teaching mathematics. The framework for deconstructing racism in mathematics offers essential characteristics of antiracist math educators and critical approaches to dismantling white supremacy in math classrooms by engaging with critical praxis in order to shift their instructional beliefs and practices towards antiracist math education.

STRIDE 2: Fostering Deep Understanding

The purpose of this tool is to highlight the diversity of student thinking, misconceptions, alternate solutions, and connections so any student, regardless of level, can contribute in meaningful discussion and gain agency and deep conceptual understanding.

STRIDE 3: Creating Conditions to Thrive

The purpose of this tool is to give teachers the resources they need to meaningfully incorporate the Aspen Institute National Commission on Social, Emotional, and Academic Development (SEAD) themes—Agency, Belonging, Discourse, and Identity—into the teaching of priority standards at grades 6-8 .

STRIDE 4: Connecting Critical Intersections

This tool was created to address the need for English learner (EL) supports in enhancing English proficiency, including language scaffolding, student talk, and clarifying their understanding of mathematical concepts. This tool seeks to assist teachers with classroom actionables in the form of scaffolding and supports that model equitable instructional practices for ELs, and foster rigor to develop language and mathematical conceptual understanding.

STRIDE 5: Sustaining Equitable Practice

This tool is a framework designed to support instructional coaches, administrators, or lead teachers who are seeking guidance on an approach to coaching that addresses equity issues in the math classroom. It focuses on developing cultural competence, emotional intelligence of both teachers and students, and the knowledge and skill-building of teaching.

Competencies for Culturally Responsive Teaching

https://www.newamerica.org/education-policy/reports/culturally-responsive-teaching/

A framework for addressing culturally responsive teaching that gathered information from fifty states outlines **eight competencies:**

Figure 1: Eight competencies for culturally responsive teaching (Source: New America)

Students come from many different backgrounds, experiences, and cultural identities. Multicultural education draws on students' experiences through their family, community, and cultural and linguistic forms of knowing, referred to as funds of knowledge (Gonzalez, Moll, & Amanti, 2006), in ways that go far beyond food, music, and folklore. A multicultural approach is foundational to participating in the global economy.

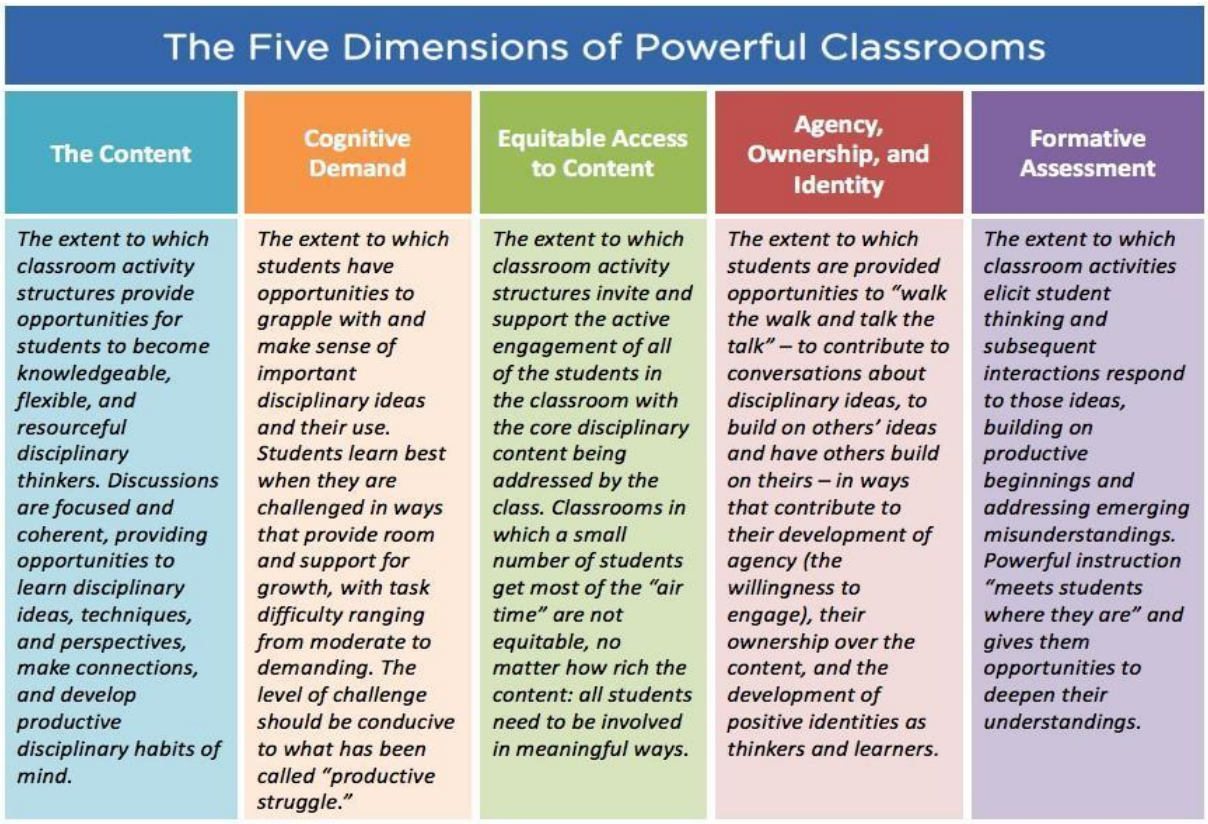
An Introduction to the Teaching for Robust Understanding (TRU) Framework

This document introduces the **T**eaching for **R**obust **U**nderstanding (**TRU**) framework. TRU provides a research-based response to the question,

> *"What are the attributes of equitable and robust learning environments – environments in which all students are supported in becoming knowledgeable, flexible, and resourceful disciplinary thinkers?"*

The answer, which resonates with what we know as teachers and researchers, appears in distilled form in figure 2. The quality of a learning environment depends on the extent to which it provides opportunities for students along the following **five dimensions:**

1. The richness of disciplinary concepts and practices ("the content") available for learning;
2. Student sense-making and "productive struggle";
3. Meaningful and equitable access to concepts and practices for all students;
4. Means for constructing positive disciplinary identities through presenting, discussing and refining ideas; and
5. The responsiveness of the environment to student thinking.



The Five Dimensions of Powerful Classrooms				
The Content	**Cognitive Demand**	**Equitable Access to Content**	**Agency, Ownership, and Identity**	**Formative Assessment**
The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.	*The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.*	*The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Figure 2: The five dimensions of powerful classrooms
Source: https://truframework.org/

Complete TRU Math Conversation Guide can be found here.

English Learner Road Map

California's Vision and Mission of Success for English Learners

The California State Board of Education unanimously approved the California English Learner Roadmap State Board of Education Policy: Educational Programs and Services for English Learners (EL Roadmap Policy) on July 12, 2017. This policy is intended to provide guidance to local educational agencies (LEAs) on welcoming, understanding, and educating the diverse population of students who are English learners attending California public schools. The California English Learner Roadmap: Strengthening Comprehensive Educational Policies, Programs, and Practices for English Learners (CA EL Roadmap) builds on the EL Roadmap Policy and provides further guidance on educating English learners. The CA EL Roadmap supports LEAs as they implement the EL Roadmap Policy.

Vision

English learners fully and meaningfully access and participate in a twenty-first century education from early childhood through grade twelve that results in their attaining high levels of English proficiency, mastery of grade level standards, and opportunities to develop proficiency in multiple languages.

Mission

California schools affirm, welcome, and respond to a diverse range of English learner (EL) strengths, needs, and identities. California schools prepare graduates with the linguistic, academic, and social skills and competencies they require for college, career, and civic participation in a global, diverse, and multilingual world, thus ensuring a thriving future for California.

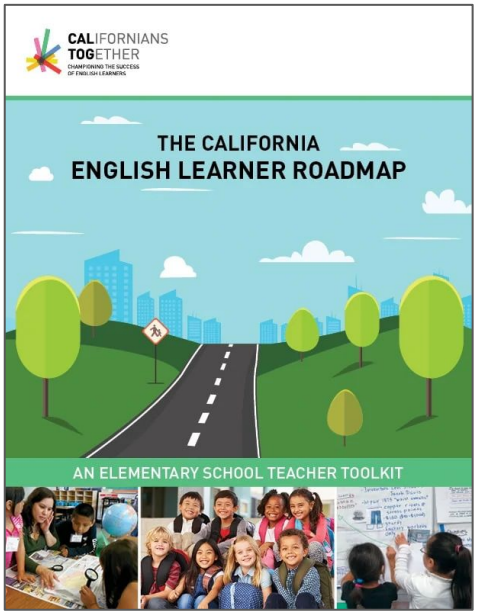
- **Policy and Printed Document:** This page contains the EL Roadmap Policy and the printable guidance document that, together with this web page, make up the *CA EL Roadmap*.
- **Research:** This page contains research related to EL program development.
- CA EL Roadmap Principles: These pages contain information on each of the principles and illustrative examples demonstrating each principle in action.
    - **Principles Overview**
    - **Principle One:** Assets-Oriented and Needs Responsive Schools
    - **Principle Two:** Intellectual Quality of Instruction and Meaningful Access
    - **Principle Three:** System Conditions that Support Effectiveness
    - **Principle Four:** Alignment and Articulation Within and Across Systems
- **Characteristics of Examples:** This page describes the characteristics of examples for inclusion in the *CA EL Roadmap*.
- **Illustrative Case Examples:** This page contains illustrative case examples from the field that illustrate the CA EL Roadmap principles and elements in action.
- **Crosswalk to LCAP:** This page contains a crosswalk between the CA EL Roadmap principles and the Local Control and Accountability Plan (LCAP) state priorities.
- **Resources:** This page contains frequently asked questions about the CA EL Roadmap and resources for LEAs and the public to use. This page includes an "At a Glance" information sheet and presentations on the CA EL Roadmap.
- **Communications and Updates:** This page contains letters and other communications on the CA EL Roadmap. This page also includes dates and locations of upcoming regional presentations and workshops.
- **Archives:** This page contains previous EL Roadmap information including workgroup members and meetings that were held.

CALIFORNIANS TOGETHER
CHAMPIONING THE SUCCESS OF ENGLISH LEARNERS

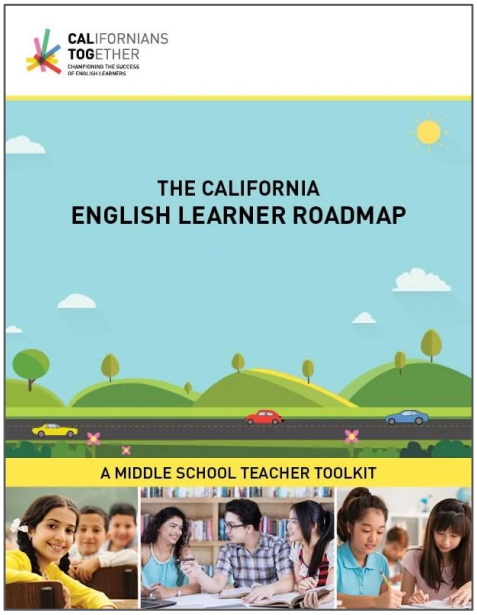
English Learner Road Map Toolkits

These toolkits are designed to help classroom teachers understand and make meaning of the California English Learner Roadmap policy. They include tools for individual and collective assessment of practices through the lens of the EL Roadmap principles, enabling teachers to hone in on their own professional learning needs and priorities. Going beyond the classroom, the tools also help teachers think about the kinds of supports and services needed in their school to support English learners. Finally, each toolkit lists resources teachers can access for their own learning and for their participation in school and district level dialogues about enhancing instructional practices and programs.

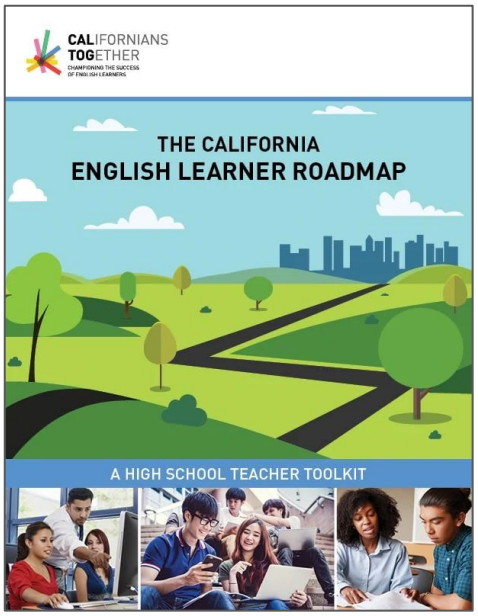
The set of EL Roadmap Teacher Toolkits includes: The Elementary Teacher Toolkit, The Middle School Teacher Toolkit, a High School Teacher Toolkit and an Administrator's Guidance Document on how to engage teachers with the EL Roadmap Toolkits.



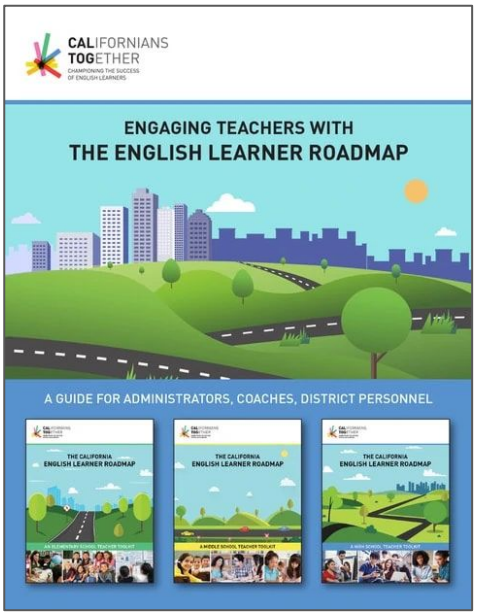
The Elementary Teacher Toolkit



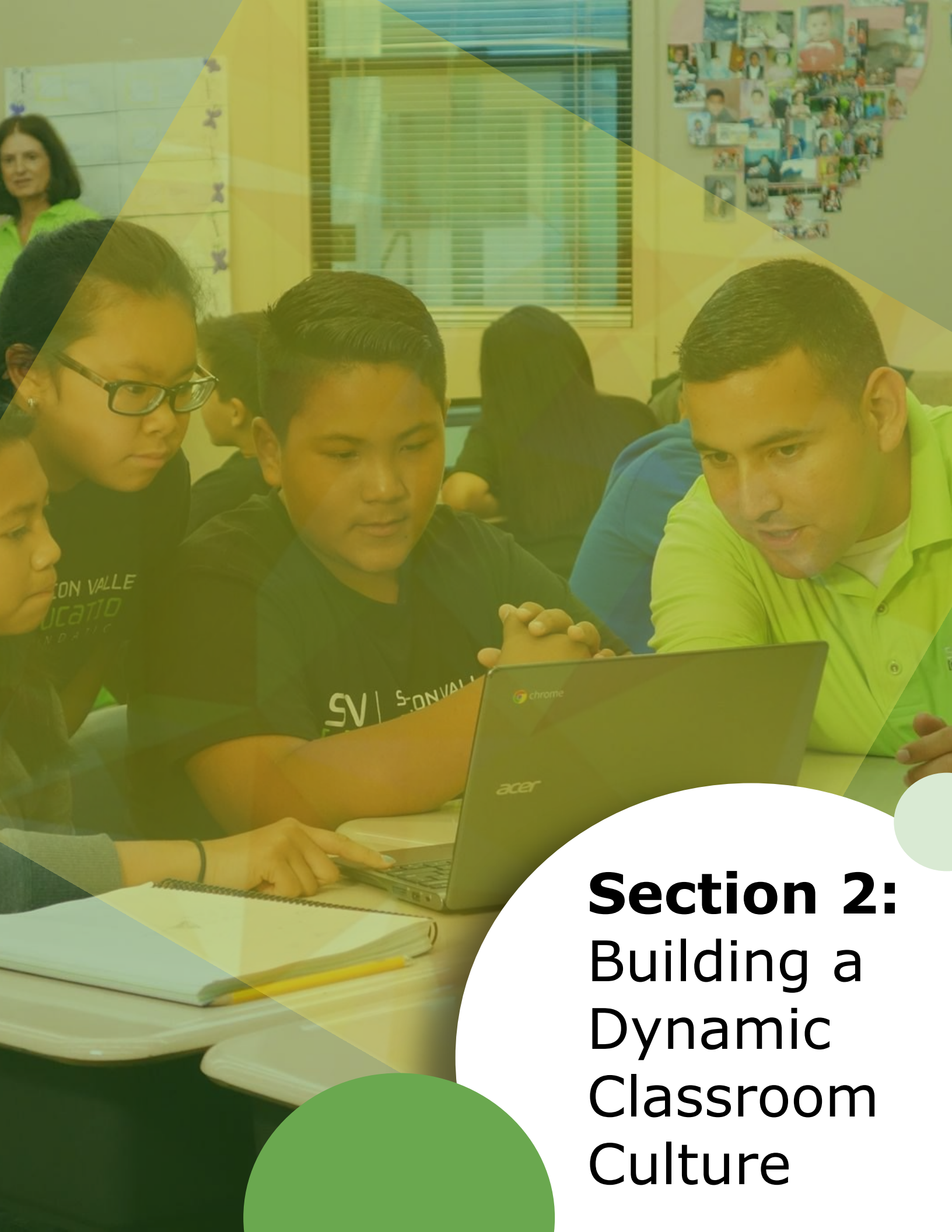
The Middle School Teacher Toolkit



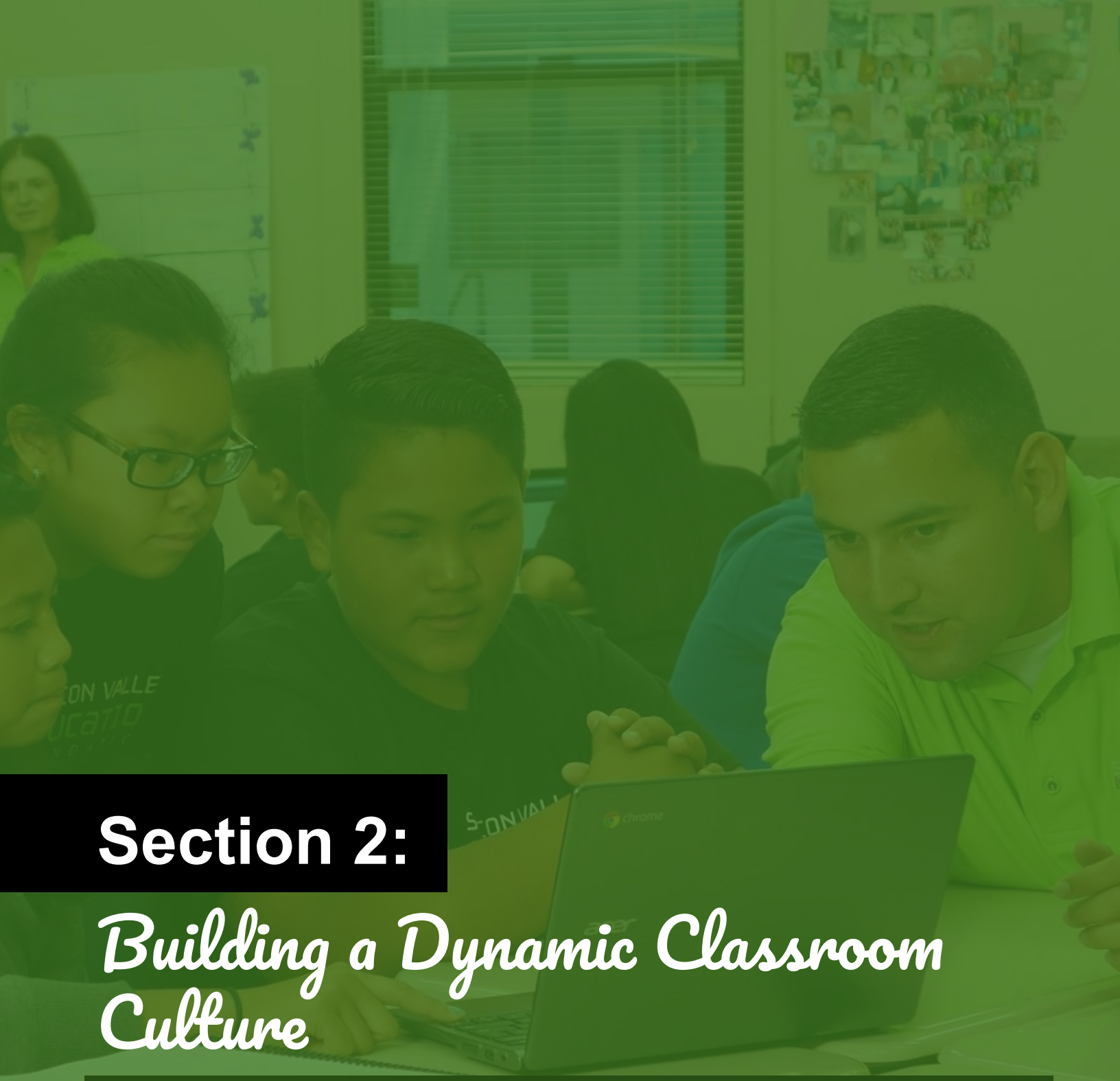
The High School Teacher Toolkit



A Guide for Administrators, Coaches and District Personnel



Section 2: Building a Dynamic Classroom Culture



Section 2:

Building a Dynamic Classroom Culture

Rich mathematics learning happens in classrooms where students are positioned as knowledge constructors and sense-makers and where teachers leverage students' experiences and ideas as the class works together to develop explanations for phenomena. Establishing and maintaining norms is an essential step in developing an equitable learning community grounded by trusting and caring relationships, a shared understanding and appreciation for diversity, and students' sense of belonging in the math classroom.

youcubed
at Stanford University

Setting up Positive Norms in Math Class

Here are 7 of my favorite messages to give to students in math class, and some suggestions from *you*cubed as to how to encourage them:

Everyone can learn math to the highest levels

Mistakes are valuable

Questions are really important

Math is about creativity and making sense

Math is about connections and communicating

Math class is about learning not performing

Depth is more important than speed

NOTE: Download the full ***"Setting up Positive Norms in Math Class by Jo Boaler"*** *here.*

youcubed
at Stanford University

Positive Norms to Encourage in Math Class

1. Everyone Can Learn Math to the Highest Levels.
Encourage students to believe in themselves. There is no such thing as a "math" person. Everyone can reach the highest levels they want to, with hard work.

2. Mistakes are valuable
Mistakes grow your brain! It is good to struggle and make mistakes.

3. Questions are Really Important
Always ask questions, always answer questions. Ask yourself: why does that make sense?

4. Math is about Creativity and Making Sense.
Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.

5. Math is about Connections and Communicating
Math is a connected subject, and a form of communication. Represent math in different forms eg words, a picture, a graph, an equation, and link them. Color code!

6. Depth is much more important than speed.
Top mathematicians, such as Laurent Schwartz, think slowly and deeply.

7. Math Class is about Learning not Performing
Math is a growth subject, it takes time to learn and it is all about effort.

*NOTE: Download the full **"Setting up Positive Norms in Math Class by Jo Boaler"** here.*

Establishing Class Norms

What is this structure?

Class norms are a set of statements of value or behavior that support active and equitable participation in the classroom. They set expectations for all members of the classroom community, adults and students. Class norms are stated in a positive tone, apply to all situations all the time, and are few in number.

Why do I use this structure?

Class norms help students and give them opportunities, to be accountable to the community and to act on the prosocial values of responsibility, respect, fairness, caring, and helpfulness. They support equitable participation, engagement with the practices of doing math, and establishment of a positive classroom climate.

When do I use this structure?

Establish classroom norms at the beginning of the school year and continue to reinforce them every day.

How do I use this structure?

Establishing class norms:

- Hold a discussion of what the community wants the classroom to look and feel like. The norms could come out of this discussion, or this discussion could lay the groundwork for discussion of teacher-chosen norms.
- It takes time and effort for students to follow the norms consistently. Provide clear expectations and examples of what norms look like. Have students brainstorm and share your own powerful vision.
- Distinguish between rules and norms. Rules, which establish classroom safety and efficiency, can support norms. Rules should be established by the teacher on the first day of school. Phrasing the rules using a positive tone contributes to a sense of community.
- Give, or have students generate, specific examples of what the norms look like and sound like.

Reinforcing class norms:

- Continue to remind and reinforce the positive behaviors that you see throughout the year. Include specific examples of what the norms look like and sound like.
- Participation Quiz and Groupwork Feedback strategies can be used when establishing the norms and, as the year continues, to support attention to the norms.
- Encourage students to notice themselves and others supporting the norms. Celebrate positive mathematical interactions.

Sample Class Norms

Choose 4-6 norms to use with your class, including one or two from each of the following value categories, depending on the needs of your students. Note that some norms support multiple values. Make the norms your own by discussing them with your students and changing the language if necessary.

Collaboration	• Respect each other. • Help each other. • Solve problems together. • Ask our group members for help before the teacher. • Talk about each other's thinking. • Ask questions until ideas make sense. • Make decisions together. • Justify your ideas. • Together, work to answer questions. • Ask questions and share ideas. • Members of your team are your first resource. • Smarter together than apart. • You have the responsibility to ask for help, and you have the responsibility to offer it. • No one is done until everyone is done.
Growth Mindset	• Errors are gifts that promote discussion. • Answers are important, but they are not the math. • Learn from mistakes. • Learn from trying new things. • Learn from taking risks.
Communication	• Ask our group members for help before the teacher. • Ask questions until ideas make sense. • Justify your ideas. • Explain and give reasons. • Ask questions and share ideas.
Persistence	• Ask our group members for help before the teacher. • Ask questions until ideas make sense. • Smarter together than apart. • You have the responsibility to ask for help, and you the responsibility to offer it. • No one is done until everyone is done.
Process	• Solve problems together. • Ask our group members for help before the teacher. • Answers are important, but they are not the math. • Talk about each other's thinking. • Ask questions until ideas make sense. • Use multiple strategies and representations. • Make decisions together. • Justify your ideas. • Together, work to answer questions. • Explain and give reasons. • Members of your team are your first resource. • You have the responsibility to ask for help, and you the responsibility to offer it. • No one is done until everyone is done.

Source: SFUSD Mathematics Department, June 2015, sfusdmath.org

Collaborative Learning Guide

Collaborative learning gives the responsibility of the learning to the students by using groups and pairs of students to fulfill a task or assignment within the classroom. The Common Core Math Practice Standard 3 calls for students at all grades to listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Creative Effective Collaborative Activities	Within a Collaborative Group
• Is the activity highly structured physically, spatially and temporally? • Do students know the rationale for the activity? • What are the teacher's academic and behavioral expectations of the students? • How will the learning activity affect motivation? • Does the activity accommodate various group speeds? • How long will the activity take? • Is there an opportunity for students to process how the group functioned and the learning that occurred during the activity? • Does the structure of the activity give students time to process the new information before they are asked to respond?	• Students are invested in their own learning. • Learners actively participate. • Teachers become learners at times, and learners sometimes teach. • Respect is given to every member. • The project / question should be of interest and challenging to the students. • Diversity is celebrated and all contributions are valued. • Students learn skills for resolving conflicts when they arise. • Members draw upon their past experience and knowledge. • Goals are clearly identified and used as a guide. • Research tools such as internet access are made available.
Possible Student Roles within the Group	**Group management Tips**
• **Facilitator:** Keeps group on task and verifies that all contribute. • **Recorder:** Takes notes on important thoughts expressed in the group. Writes final summary. • **Reporter:** Shares summary of group with large group. Speaks for the group, not just personal view. • **Materials Manager:** Picks up, distributes, collects, turns in, or puts away materials. • **Time Keeper:** Keeps track of time and reminds group how much time is left.	• **Noise:** Develop and practice a "QUIET or Zero-noise" signal. Brainstorm what that would be with the students. Practice appropriate internal and external voices. • **Deadlines and Task Structure:** Give students specific tasks to finish within a predetermined time limit. Use a timer. • **Instructions:** Show, don't tell, instructions (have a group model the steps). Have students tell each other the instructions to make sure they understand prior to starting the task. • **Questions:** Answer team questions only. Individual questions should be handled within the team. Use the "3 Then Me" technique. • **Circulate:** Use proximity. Monitor discussions to check for understanding and to be aware of collaborative skills that may need to be addressed.

Source: Excerpts from "Speaking and Listening Learning Stations, Grades 6 – 8" by Schyrlet Cameron and Suzanne Myers

Remote Learning Resource

How do we build and support norms in classrooms?

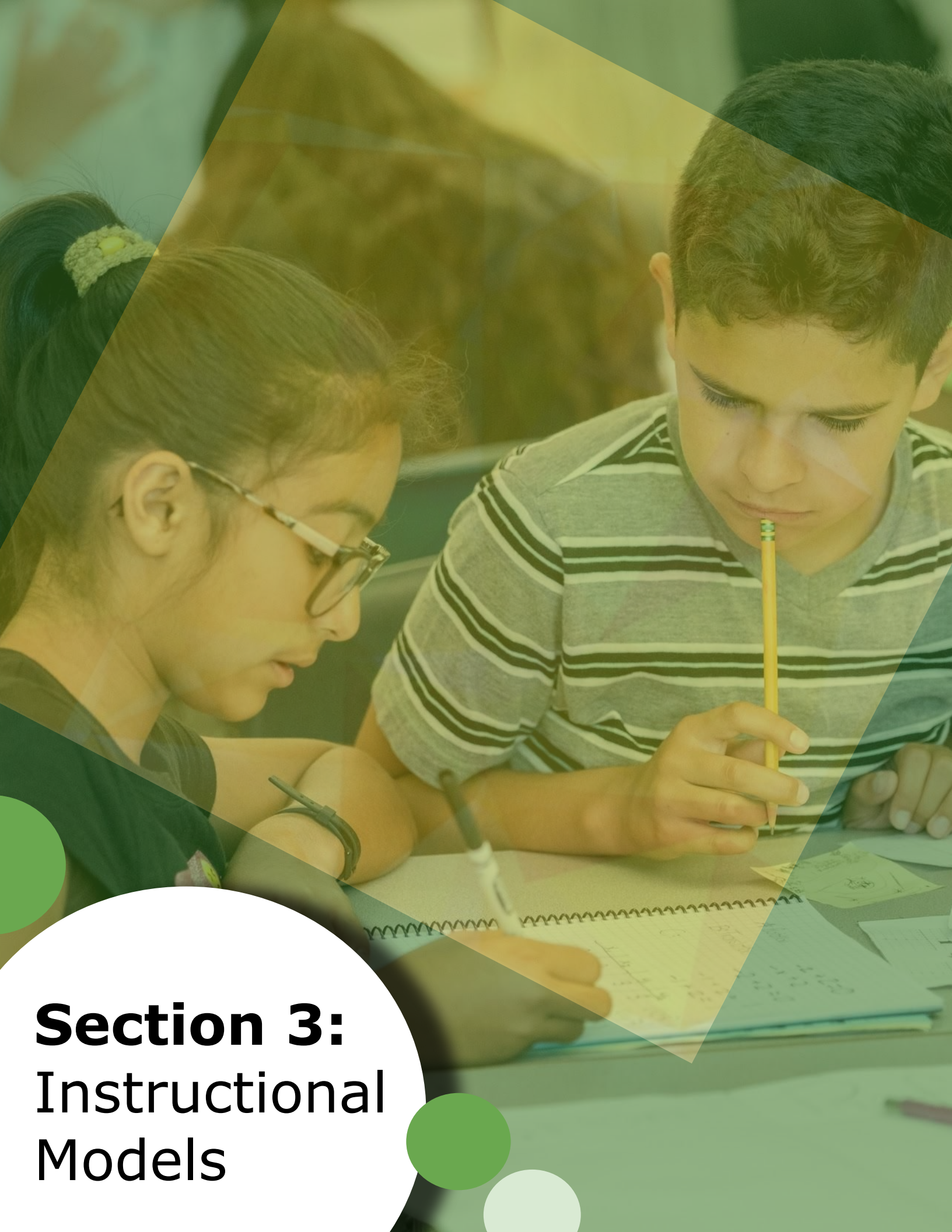
The **OpenSciEd** instructional model, the inquiry Hub instructional model, and the Next Gen Story lines instructional model all support the early co-construction of and frequent revisiting of classroom norms that allow students to see themselves as knowers and doers of science. Teachers and students work together to generate a vision of an equitable classroom by agreeing to actions that

- are respectful and make classrooms safe places for sharing;
- are equitable so that everyone's ideas and participation are valued;
- support commitment to community and learning together; and
- move math thinking forward as students work to figure things out.

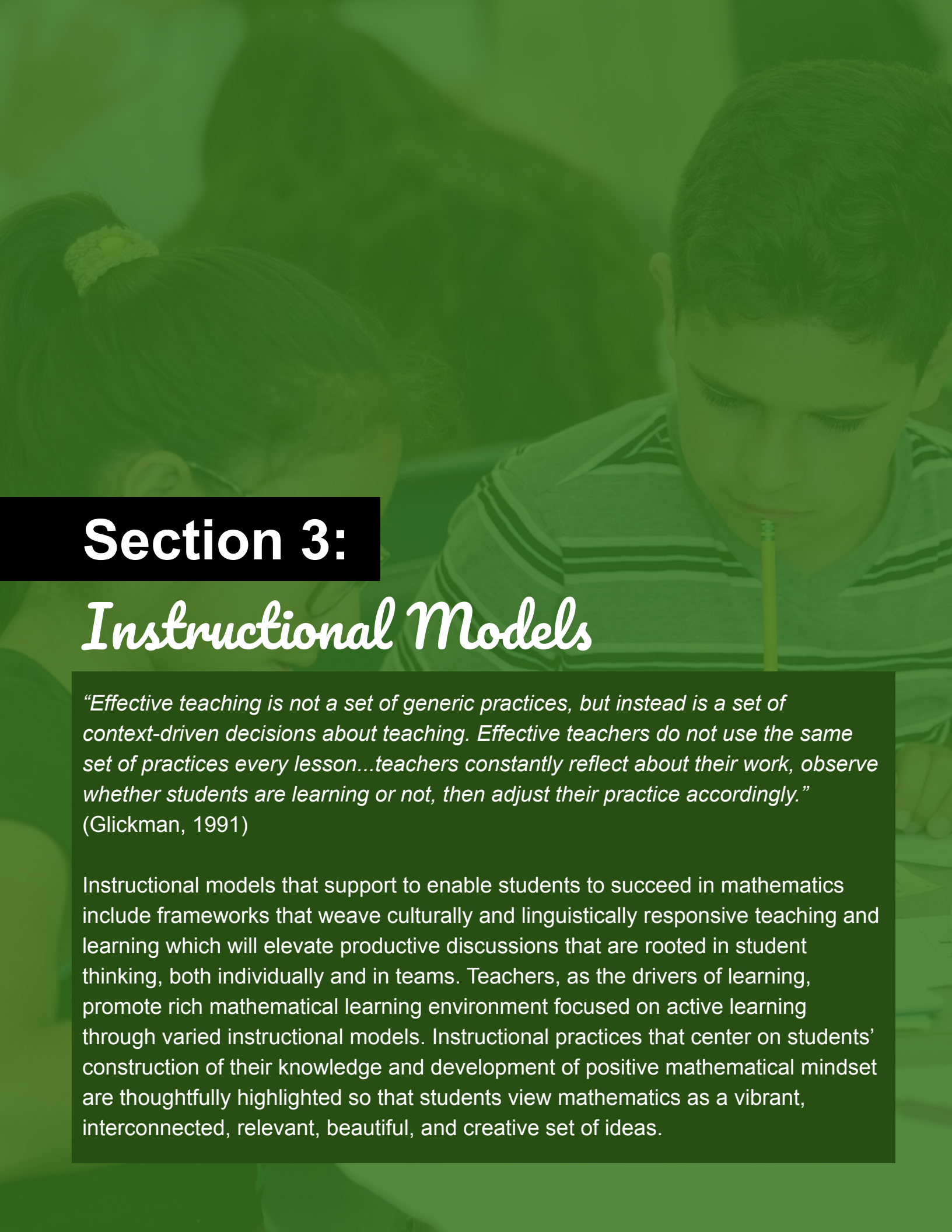
How can a community care for one another when they can't meet in person?

Creating norms in a face-to-face classroom setting often begins with asking the students to describe and agree on actions that reflect the ideals described above. This practice can be useful in a remote environment, but must also attend to the features and constraints of the technology used (or not used). When remote learning is new or necessitated by circumstance, contextualizing familiar norms may help students and teachers feel more at ease. In your efforts to establish norms for remote learning, remember that learning can be messy under the best circumstances. Be flexible and remember that perfection is not the goal. Although all people need to express and receive care from one another, acts of care take different forms. Care can involve giving help to a student who needs it. An educator can show care by taking everyone's ideas seriously. It is evident when students feel they are treated fairly and when they are trusted. When working at a physical distance from students, consider different ways to reach out - by phone, text, letter, email, or over video-conference - to express care.

*NOTE: Download the full **"Remote Learning Source"** here.*



Section 3: Instructional Models



Section 3:

Instructional Models

"Effective teaching is not a set of generic practices, but instead is a set of context-driven decisions about teaching. Effective teachers do not use the same set of practices every lesson...teachers constantly reflect about their work, observe whether students are learning or not, then adjust their practice accordingly." (Glickman, 1991)

Instructional models that support to enable students to succeed in mathematics include frameworks that weave culturally and linguistically responsive teaching and learning which will elevate productive discussions that are rooted in student thinking, both individually and in teams. Teachers, as the drivers of learning, promote rich mathematical learning environment focused on active learning through varied instructional models. Instructional practices that center on students' construction of their knowledge and development of positive mathematical mindset are thoughtfully highlighted so that students view mathematics as a vibrant, interconnected, relevant, beautiful, and creative set of ideas.

Culturally and Linguistically Responsive Teaching and Learning

Based on the work of Dr. Sharroky Hollie

Culturally and linguistically responsive teaching and learning (CLR) is focused on improving instruction for underserved students (Hollie, 2018). Four features define the key aspects of CLR: validating, affirmation, build, and bridge (VABB).

- **Validation** is the intentional and purposeful legitimization of the home culture and language of the student.
- **Affirmation** is the intentional and purposeful effort to reverse the negative stereotypes, images, and representations of marginalized cultures, languages promoted by corporate mainstream media.
- **Building** is understanding and recognizing the cultural and linguistic behaviour of students and using those behaviours to foster rapport and relationships with the students.
- **Bridging** is providing the academic and social skills that students will need to have success beyond your classroom.

Watch "Dr. Hollie Q&A" to know more about VABB.

Watch "CLR Instructional Strategies (3rd Grade Math)"

5 Practices for Orchestrating Productive Mathematics Discussions

Based on the book by Margaret Smith and Mary Kary Stein

Facilitating productive discussions about mathematics is very challenging for any teacher. Some lessons can end effectively with a “share and summarize.” At other times, though, a more purposeful discussion is needed to bring out the key mathematics of a lesson. A key component of productive discussion is teacher facilitation. This facilitation is not accidental and cannot, generally, happen on the fly.

Here are **5 concrete steps** that can help improve the quality of mathematics discussion in your class.

1. **ANTICIPATING likely student responses to mathematical tasks.**
    - Do the problem yourself
    - What are students likely to produce?
    - Which problems will most likely be the most useful in addressing the mathematics?

2. **MONITORING students’ actual responses to the tasks.**
    - Listen, observe, identify key strategies
    - Keep track of approaches
    - Ask questions of students to get them back on track or to think more deeply

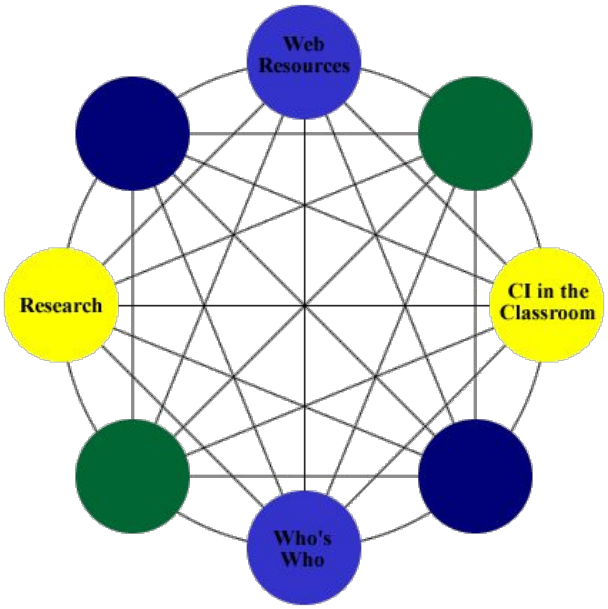
3. **SELECTING student response to feature during the discussions.**
    - CRUCIAL STEP - what do you want to highlight?
    - Purposefully select those that will advance mathematical ideas

4. **SEQUENCING student responses during the discussions.**
    - In what order do you want to present the student work samples?
    - Do you want the most common? Present misconceptions first?
    - How will students share their work? Draw on board? Put under doc cam?

5. **CONNECTING student responses during the discussions.**
    - Craft questions to make the mathematics visible.
    - Compare and contrast 2 or 3 students’ work - what are the mathematical relationships?
    - What do parts of student’s work represent in the original problems? The solution? Work done in the past?

Stanford | GRADUATE SCHOOL OF EDUCATION

Complex Instruction



Complex Instruction evolved from over 20 years of research by Elizabeth Cohen, Rachel Lotan, and their colleagues at the Stanford School of Education. **The goal of this instruction is to provide academic access and success for all students in heterogeneous classrooms.**

Complex Instruction (CI) has three major components:

1. Multiple ability curricula are designed to foster the development of higher-order thinking skills through groupwork activities organized around a central concept or big idea. The tasks are open-ended, requiring students to work interdependently to solve problems. Most importantly, the tasks require a wide array of intellectual abilities so that students from diverse backgrounds and different levels of academic proficiency can make meaningful contributions to the group task. Research has documented significant achievement gains in classrooms using such curricula.

2. When using special instructional strategies, the teacher trains the students to use cooperative norms and specific roles to manage their own groups. Then, the teacher is free to observe groups carefully, to provide specific feedback, and to treat status problems which cause unequal participation among group members.

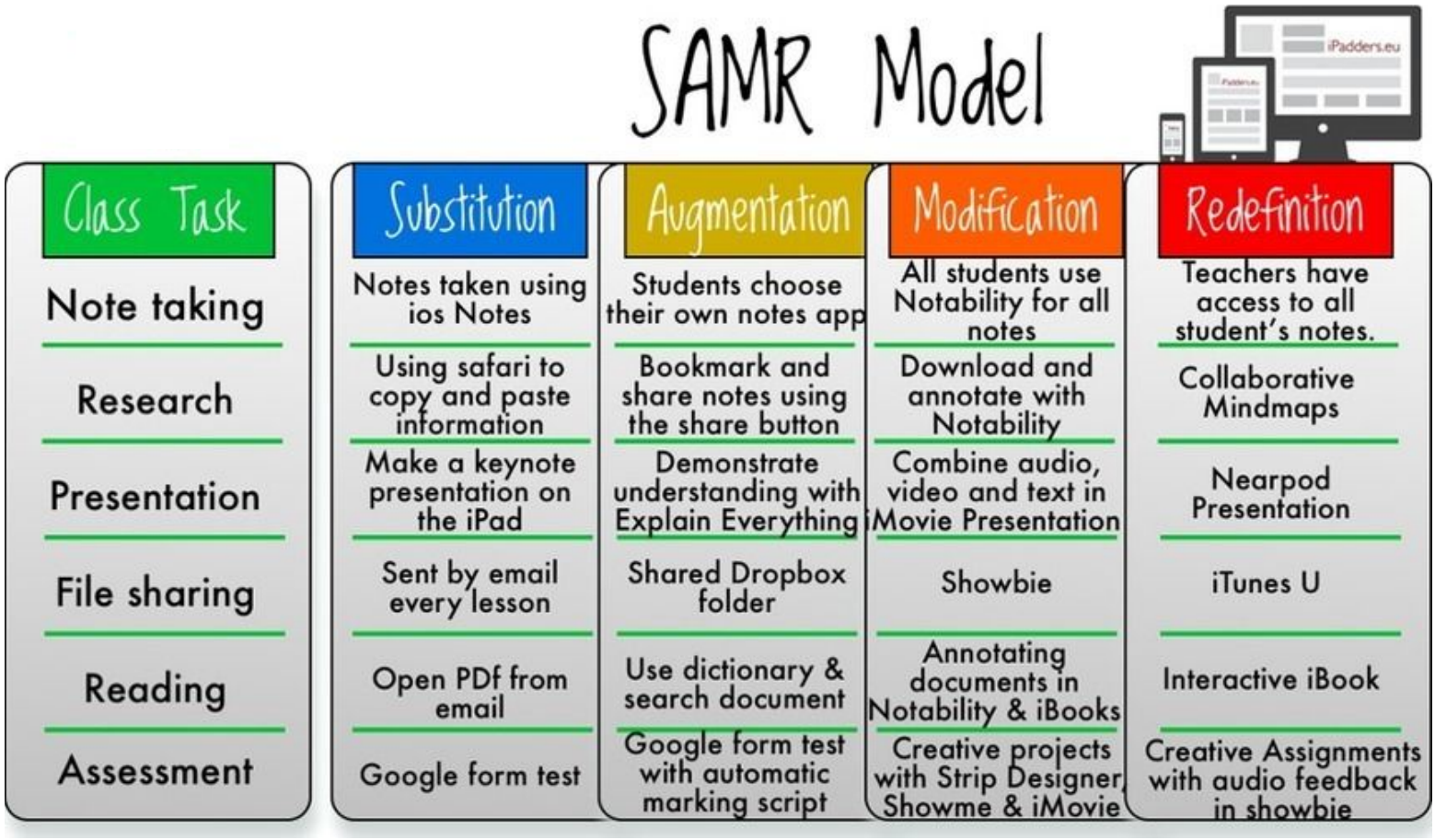
3. To ensure equal access to learning, teachers learn to recognize and then treat status problems. Sociological research demonstrates that in CI, the more that students talk and work together, the more they learn. However, students who are social isolates or students who are seen as lacking academic skills often fail to participate and thus learn less than they would if they were more active in the groups. In CI, teachers use status treatments to broaden students' perceptions of what it means to be smart, and to convince students that they each have important intellectual contributions to make to the multiple-ability task.

Source: https://complexinstruction.stanford.edu/

Technology Integration
SAMR Model

The SAMR Model is a framework created by Dr. Ruben Puentedura that categorizes four different degrees of classroom technology integration. The letters "SAMR" stand for Substitution, Augmentation, Modification, and Redefinition. The SAMR model was created to share a common language across disciplines as teachers strive to help students visualize complex concepts.

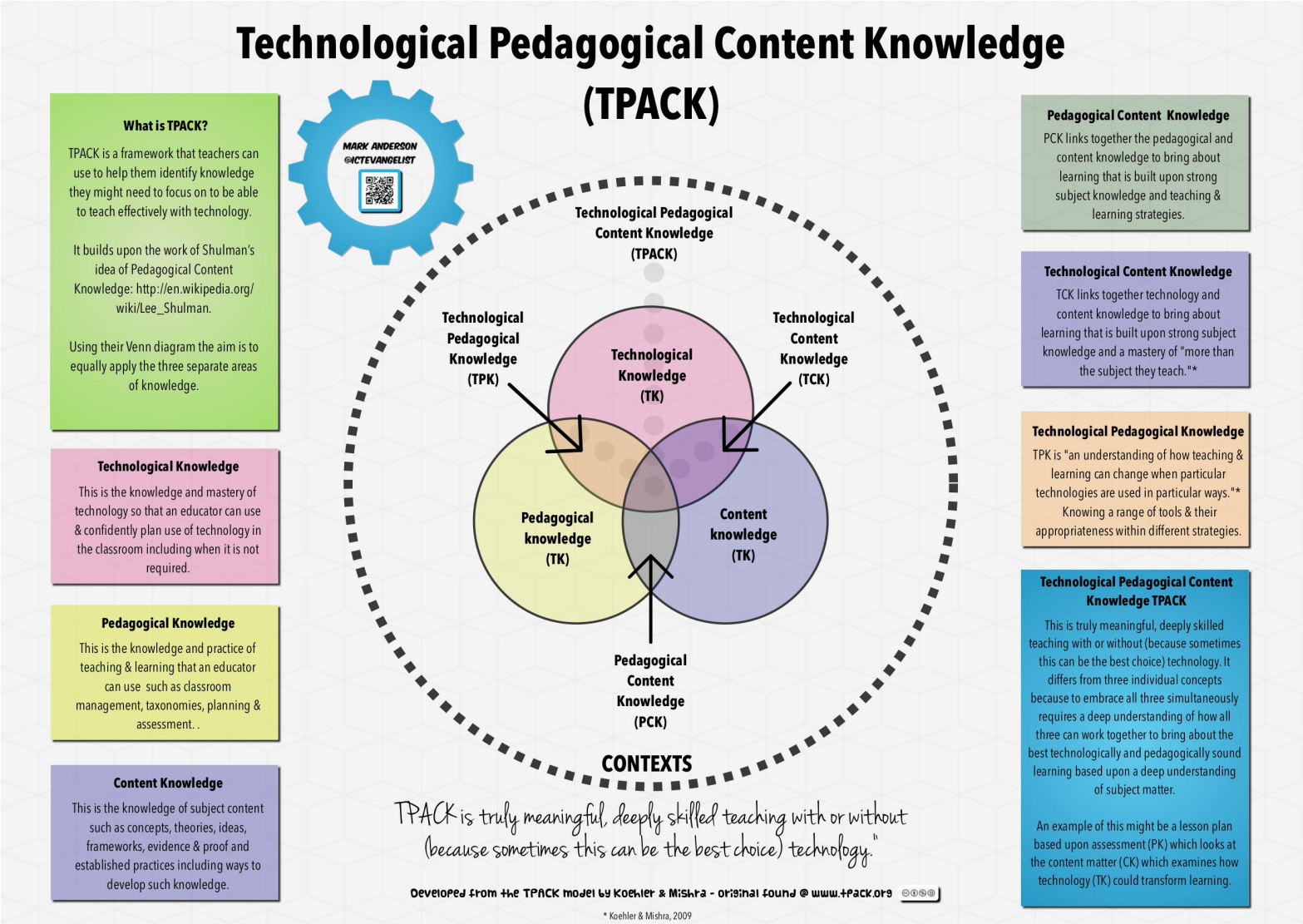
Watch "What is the SAMR Model and what does it look like in schools?" by John Spencer?

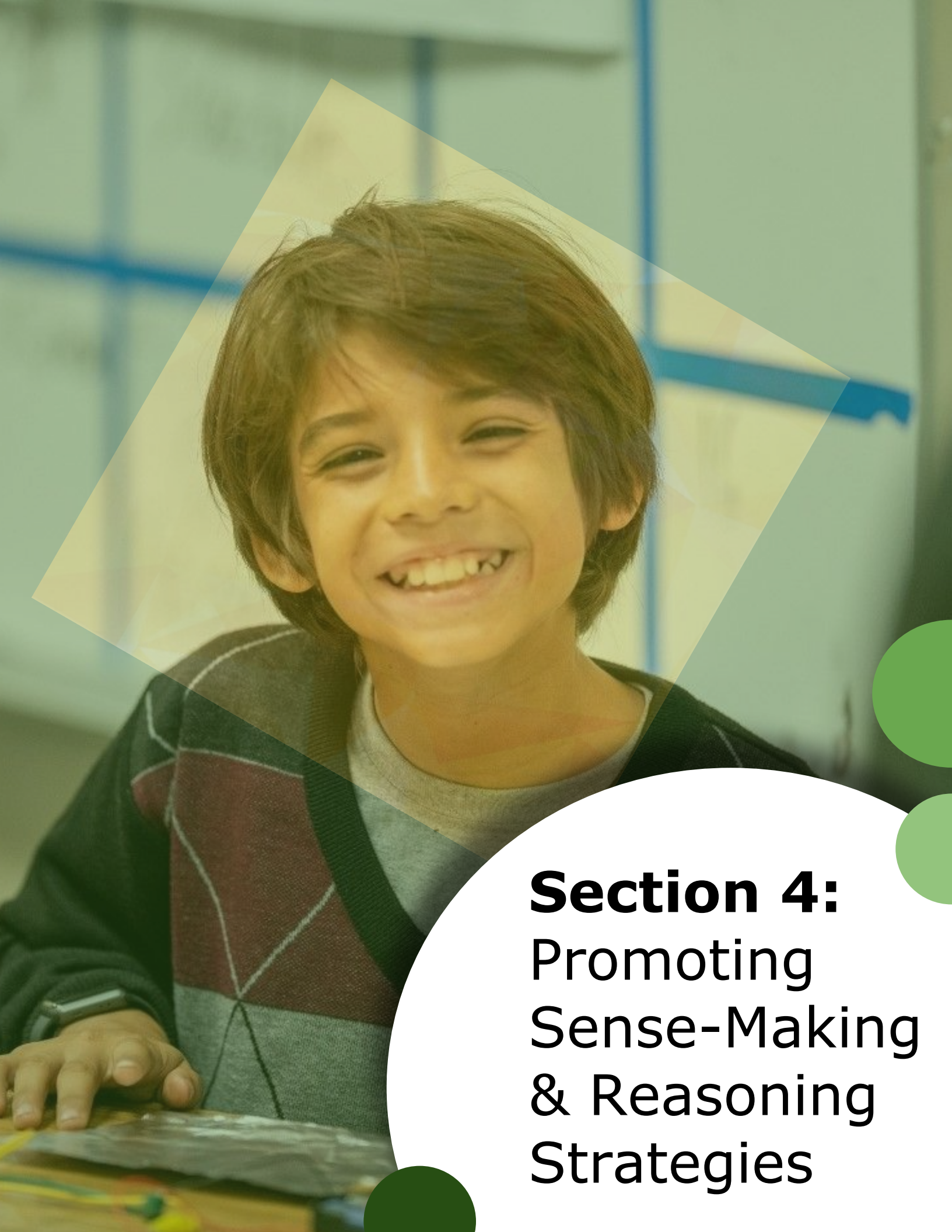


Technology Integration
Technological Pedagogical Content Knowledge (TPACK)

The TPACK model addresses the interaction of technological, pedagogical, and content knowledge and how they relate to teaching in a technology-enhanced learning environment. TPACK, is a useful model for educators as they begin to use digital tools and strategies to support teaching and learning. This model, developed by educational researchers Mishra and Kohler (2006), is designed around the idea that content (what you teach) and pedagogy (how you teach) must be the basis for any technology that you plan to use in your classroom to enhance learning.

Watch *"Introduction to the TPACK Model Video"* by Common Sense Media





Section 4: Promoting Sense-Making & Reasoning Strategies

Section 4:

Promoting Sense-Making & Reasoning Strategies

The overarching focus to promote sense-making and reasoning as critical pedagogical approaches, while blending the Common Core State Standards (CCSS) with engaging tasks gives students opportunities to engage in quality discussions within a mathematical context.

“Reasoning and sense making are the foundation of mathematical competence and proficiency, and their absence from the curriculum leads to failure and disengagement in mathematics instruction. Thus, developing students’ reasoning and sense-making capabilities should be the primary goal of mathematics instruction. In order to achieve this goal, all mathematics classes should provide ongoing opportunities for students to implement these processes. Reasoning and sense making are critical in mathematics learning because students who genuinely make sense of mathematical ideas can apply them in problem solving and unfamiliar situations and can use them as a foundation for future learning” *(Mathematical Reasoning and Sense Making in Grades 6-8 by Michael T. Battista)*.

Technology Tools

Flipgrid is a simple, free, and accessible video response platform where educators can have online video discussions with students or other educators.

Google Slides is a presentation program that allows users to create and edit files online while collaborating with other users in real-time.

Jamboard is an interactive whiteboard system developed by Google and has compatibility for online collaboration through cross-platform support through the free Google Docs suite.

Nearpod is a student engagement platform built to make teaching with technology easy. With Nearpod you can control what your students see and get feedback in real-time.

Padlet is a free online tool that is best described as an online notice board. Padlet can be used by students and teachers to post notes on a common page.

Pear Deck is an interactive presentation tool used to actively engage students in individual and social learning. Teachers create presentations using their Google Drive account

Seesaw is a student-driven digital portfolio that empowers students to independently document what they are learning at school.

Whiteboard.fi is a digital whiteboard and an instant formative assessment tool for your classroom, providing you with live feedback and immediate overview over your students.

Zoom Annotation allows you to draw on a shared screen, and Whiteboard allows you to write on a blank screen everyone can see.

“Intentionally encouraging students to share ideas, solutions, and justifications promoted strategies for discussion.”

Middle School Teacher

Toolkit Strategy:

Math Talks

Toolkit Strategy
Math Talks

SMP 2: Reason abstractly and quantitatively.
SMP 3: Construct viable arguments and critique the reasoning of others
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: TK - 12

Purpose:

- Develop quality student discourse in a whole class setting, with improved problem solving and mathematical thinking.
- Encourage student thinking, justifying reasoning, and making sense of other's strategies with precision in explanations and mathematical reasoning.
- Gain an understanding of numbers and math concepts by improved precision, reasoning and exploring others' responses, backed by validation and critical thinking for mathematical challenges by verbal exchanges.
- Acquire new strategies to problem solve and increase computational fluency.
- Learn to incorporate mistakes to support risk taking, foster a growth mindset when problem solving, and build confidence and motivation when attempting mathematical challenges.

Description:

- An environment where students are able to foster mathematical thinking, to value each student's thinking, to exchange ideas and information, and actively listen and critique each others reasoning is needed.
- The teacher steps away from the role of authority, and takes the role of a facilitator - by asking students questions, recording student responses on the board.

When:

- Beginning of math class
- During classroom discussions.
- When students are providing feedback during think-pair-share.

Why:

- To build number sense.
- To engage students as active listeners.
- To reflect on students' own understanding and others' mathematical thinking and knowledge.
- Foster a growth mindset.

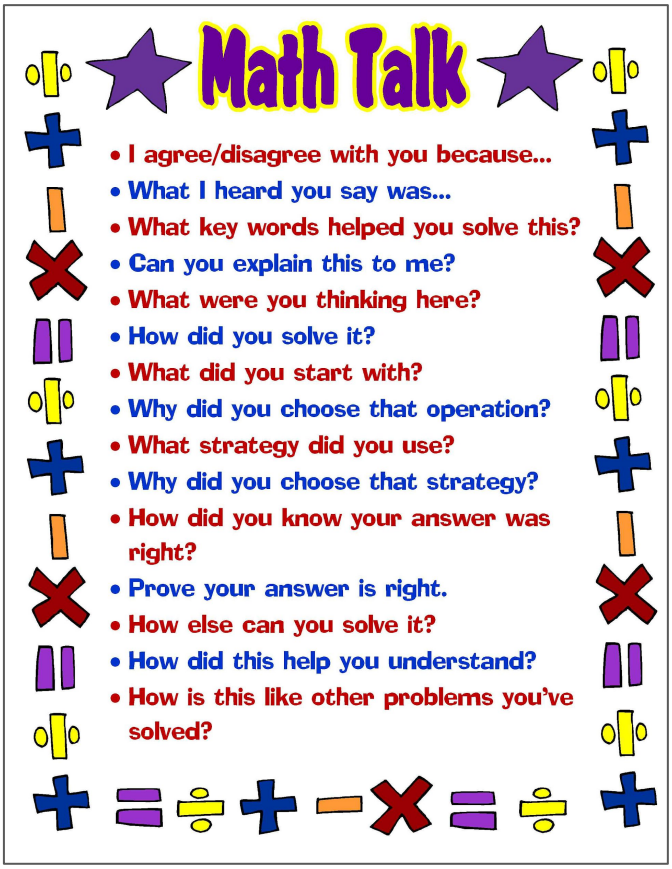
How:

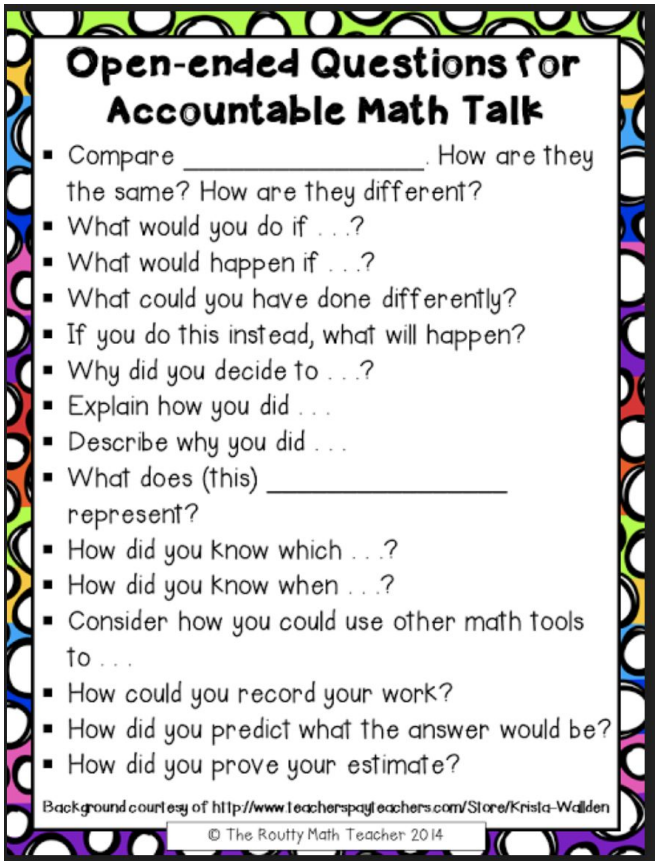
- Teacher launches a mathematics problem and presents it to students visually and orally (for virtual learning teachers can share their screen).
- Students are given sufficient independent, silent time to think, process, and come up with a solution and as many strategies as they can to respond to the question and be able to explain their reasoning.
- Students wait until the teacher gives the signal that time is up (Silent signals can be established as part of classroom routine).
- Teacher may ask students to share their answers with an elbow partner prior to sharing with the whole class.
- Teacher asks for volunteers to share their answer and strategy with the whole class, records each strategy (even if incorrect) on the whiteboard/shared screen. Teacher does NOT correct yet, just scribes the strategies shared by students.

Toolkit Strategy
Math Talks (cont.)

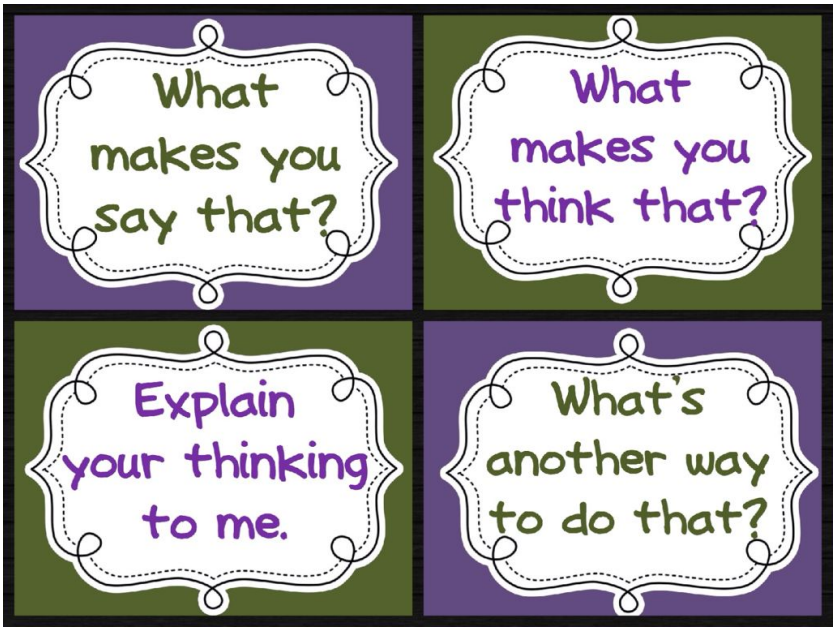
- Teacher now asks explanatory questions using prompts such as:
  - Tell me more about why you think ….
  - How did you figure that out?
  - Did anyone else get that answer, but for a different reason?
  - Can anyone come up with a justification for ____?
- Teacher can now ask questions that connect students thinking, based on what was shared out, using prompts such as:
  - What questions do you have for _____?
  - Who can paraphrase what _____ is saying?
  - Who can explain what _____ is thinking?
  - Do you agree or disagree with what they said? Can you explain why?
  - Point out similarities and differences between different strategies.
- Teacher can then summarize by asking students to point out similarities and differences between different strategies discussed.
- Math Talks should take approximately 10-15 minutes total (it may take more time initially as students learn the routine and expectations).

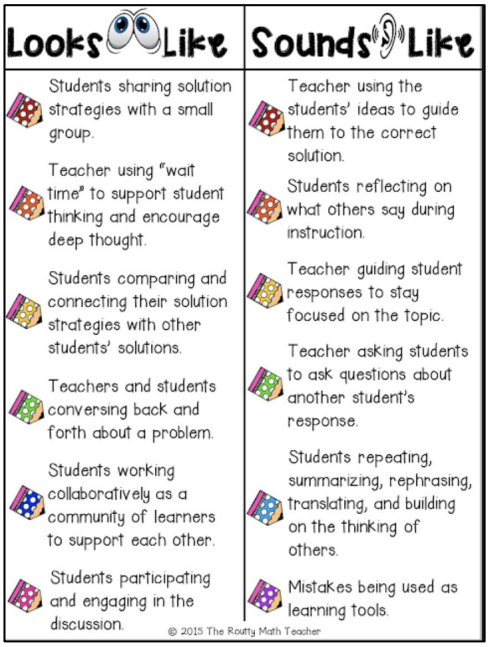
Examples *(Credit: Scholastics)*:





Toolkit Strategy
Math Talks (cont.)





Additional Resources:

- Number Talk- Jo Boaler
- Number Talks- How to do it in elementary grades- YouCubed
- Number Talks -Grade 8
- Number Talks

Lesson Plan Templates:

- YouCubed Lesson Plan
- Math Talk for High school
- Math Talk lesson Plan Template
    - Math talk Short Version Template
    - Math Talk Rubric

Variations to Math Talks:

- **Visual patterns**
- **Estimation**
- **Math Talk lessons**
- **Graphing Scenarios**
- **Math Errors**
- **Open Middle**
- **Which one doesn't belong?**
- **Math Talk on Fractions**
- **Number Talk**

Distance Learning Tools: Flipgrid, Whiteboard.fi, Google Slides, Jamboard, Padlet

“Multiple Representations tasks facilitate discourse among my students to build a shared understanding of mathematical ideas by analyzing and comparing their approaches and arguments.”

Elementary School Teacher

Toolkit Strategy:

Multiple Representations

Toolkit Strategy
Multiple Representations

SMP 1: Make sense of problems and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 4: Model with mathematics.

Grade Span: 3 - 12

Purpose:
- Supports students in thinking fluidly about the mathematical concepts and focuses on meaning making.
- Shows thinking through multiple representations helps students have a stronger and deeper understanding of mathematics.
- Allows students to see connections across concepts and topics.

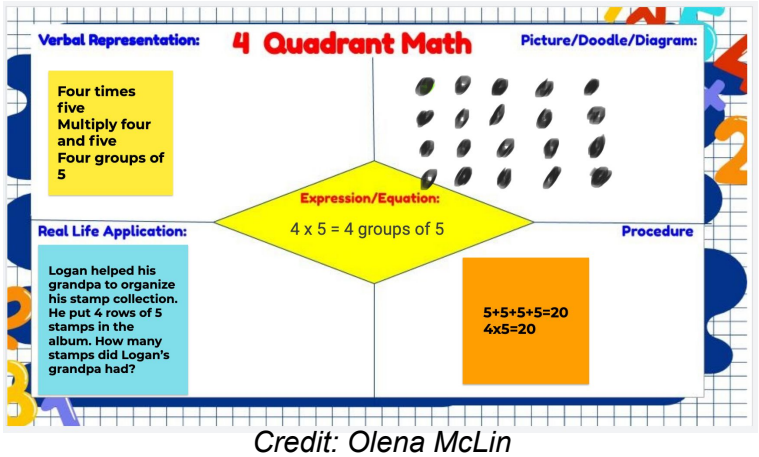
Description:
- Students create a mathematical model, apply methods, draw diagrams, connect ideas, reason about connections, and communicate their thinking.
- Students use a Multiple Representation template (also called a “4 Quadrant Math” template) to make sense of a problem and represent their thinking in multiple ways.
- The Multiple Representations template is adapted from YouCubed.

When:
- Multiple Representations can be used throughout a math lesson for most mathematical problems.

Why:
- Allows multiple entry points for solving problems, so students can think through math from different lenses
- Students dive deeper into the mathematics
- Great way to help students organize their thinking
- Practice the use of Math language and SMPs

How:
- Pose an equation, or an expression for students to work on.
- Allow students time to read, analyze and represent the equation/expression, etc. posted
- Encourage multiple representations such as writing an equation symbolically or in context, use visuals for representation, or give an example where the expression or equation is used in real-life.
- Analyze students’ work during varying stages of instruction.

Examples:

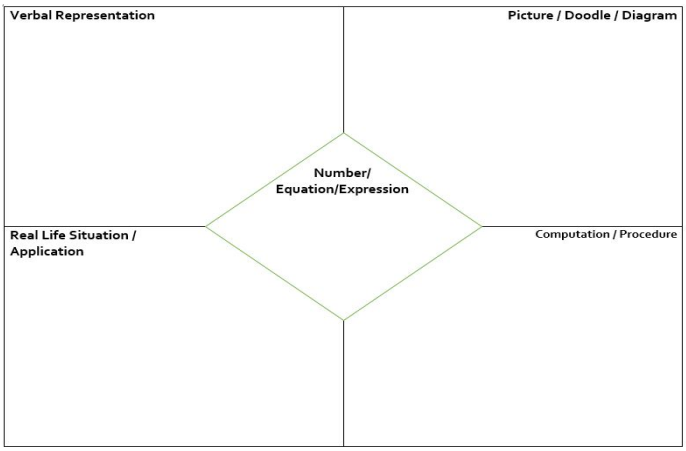


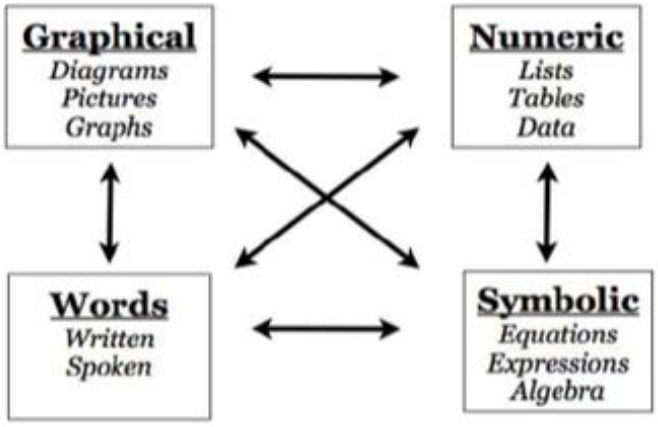
Credit: Olena McLin

Toolkit Strategy
Multiple Representations (cont.)

Additional Resources (Variations):

- Teachers can begin by providing the information for any of the sections of the template and then ask students to solve for the other sections.
    - For example, provide the real life situation/application and ask students to complete the verbal representation, a picture or a doodle, computation/procedure, and an expression/equation.





- Multiple Representations Template & Samples
    - Google Slides
    - Google Doc (Grade 3)
    - Google Doc (Generic Template)
- Blog- Mr Tippin
- Blog- Making in Math

Distance Learning Tools: Jamboard, Google Slides, Nearpod, Pear Deck, Padlet, Virtual Journal

“This has been insightful for me. I have consciously tried to make my questions open ended, deeper and richer.”

Middle School Teacher

Toolkit Strategy:

Quality Questioning

Toolkit Strategy
Quality Questioning

SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- Teachers engage students in both cognitive and metacognitive operations to develop deep understanding and long-term retention of content.
- Students ask academic questions to clarify or extend understanding and contribute positively to a learning space in which thinking, asking questions, and justifying are valued.

Description:
- Use specific questions to probe critical thinking and deeper meaning conversations to encourage further questioning and develop problem-solving skills.

When:
- When students are working on a task, the teacher circulates and asks questions to clarify student thinking.
- During the different stages of instruction, questions are posed to facilitate discourse.
- When students ask questions with one another while working together.

Why:
- Quality questions serve as a guiding inquiry to deepen and expand students' mathematical thinking and problem-solving abilities.
- Thinking through quality questioning develops student response-ability by using structural supports for thinking and reasoning.

How:
- During discussions, teachers can probe and redirect, and encourage students to interact with other students
- Students will pay attention to all questions and answers, think of answers to all questions, use wait times to think about answers, give wait time to others when asking questions, ask questions when confused and when curious, and make meaning out of facts.

Examples:
- **Lesson Plan:**
  - Achieve the Core Lesson Planning Tool
- **Lesson Plan Template:** Template 1 | Template 2 | Template 3 | Template
  - *Credit: Adapted from Math Solutions*

Additional Resources:
- What is an Essential Question? (video)
- Learning Targets and Essential Questions (video)
- Inquiry-Based Learning: Developing Student-Driven Questions (video)

Variations:
- Questions are tiered based on student needs.
- Questions are based on where students are in their problem solving process.
- Students critique the reasoning of their peers- when working as a group and during review.

Distance Learning Tools: Jamboard, Google Slides, Nearpod, Pear Deck, Padlet, Virtual Journal

Toolkit Strategy

Quality Questioning (cont.)

Examples of Quality Questions

Getting Started	• What is this problem about? • What do you (need to) know? • How can you get the information? • What do you need to find out? • What terms do you understand or not understand? • Have you solved similar problems that would help?
While Working	• What have you already tried? • How can you organize the information? • Can you make a drawing/model to explain your thinking? • What would happen if…? • Can you make a prediction? • Are there other possibilities? • Do you see any patterns or relationships that might help you solve this? • Can you describe an approach/strategy you can use to solve this? • What do you need to do next? • Why did you…? • How did you…? • What did you…? • How does this relate to…? • What assumptions are you making? • Is there another way to (draw, explain or say) that?
Reflecting About the Solution	• How do you know your solution/ conclusion is reasonable? • How do you know you have solved the problem? • How did you arrive at your answer? • How can you convince me that your answer makes sense? • What did you try that did not work? • Is that the only possible answer? • How can your explanation be made clearer?
Responding to Help Clarify Thinking	• Tell me more about your approach. • Can you explain that in a different way? • Is there another possibility or strategy that would work? • Would this method work in other problems with different numbers? • Help me to understand this part… • Is there another strategy that would work? • How does the mathematics in this problem relate to the mathematics in this unit? In previous units? • What from this problem, might you add to your notebook? • What question would you ask your teacher?

The Art of Questioning in Mathematics

Question frames to guide inquiry, deepen mathematical thinking and problem-solving skills.
(Source: NCTM Professional Teaching Standards)

Guiding questions during group work to make sense of mathematics	• Who agrees/ disagrees? Who will explain why or why not? • Who has the same answer, but a different way to explain it? • Who has a different answer? What is your answer and how did you get it? • Please ask the rest of the class that question. • Explain to your partner your understanding of what was just said. • Convince us that makes sense.
Guiding questions to reason mathematically	• Does that always work? Why or why not? • Is that true for all cases? Explain. • What is a counterexample for this solution? • How could you prove that? • What assumptions are you making?
Guiding questions to conjecture, invent, and solve problems	• What would happen if _____? What if not? • Do you see a pattern? Explain. • What about the last one? • What did you think about the problem? • What decision do you think he/she should make? • What is alike and what is different about your method of solution and his/hers? Why?
Guiding questions to connect the mathematics, the ideas, and the applications	• How does this relate to _____? • What ideas that we have learned before were useful in solving this problem? • What problem have we solved that is similar to this one? • How are they the same? How are they different? • What uses of mathematics did you find in the newspaper last night? • What example can you give me for _____?

“Math is all about Number sense-- Students who are made to see that way enjoy Math! Everyday we use numbers and so there is ample opportunity to see and wonder about the enormous impact numbers make in developing math skills.”

Middle School Teacher

Toolkit Strategy:

Number Sense

Number Sense: Visual Routines

Toolkit Strategy
Number Sense: Visual Routines

SMP 4: Model with mathematics.
SMP 7: Look for and make use of structure.
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: TK - 12

Purpose:
- Visualize and help conceptualize quantities
- Develop understanding of quantity
- Give meaning to numbers
- Understand and see relationships of numbers
- Understand the operation of numbers

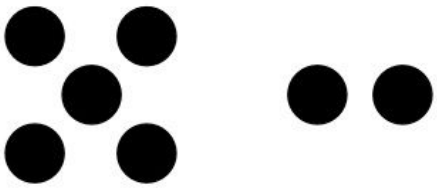
Description:
- Teachers engage students in routines that help build number sense, numerical literacy, and understanding of numbers.
- These number sense routines are not “auto pilot” activities, but opportunities for meaningful practice.

When:
- Visual Routines can be used at any time during a math lesson, but generally are done at the beginning of the lesson.

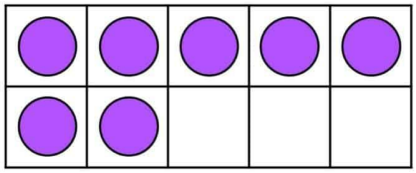
Why:
- Include discussion about numbers and their relationships
- Build on students’ existing number sense
- Encourage students to play with numbers and enrich their mathematical thinking
- Help students make connections to big ideas in mathematics

How:

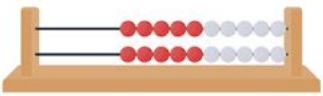
1. **Quick Images Using Dot Cards:**
   - Use images of dots based on doubles, twos, fives, tens, or a similar arrangement to those on dice or dominoes
   - Quickly show the dot card for 3-5 seconds so students can visualize the amount of dots
   - Ask students what they saw, encouraging them to think in groups as opposed to counting by ones



2. **Ten Frames:**
   - Show an image of a ten frame with dots to represent a number
   - Ask students what number is represented by the ten frame



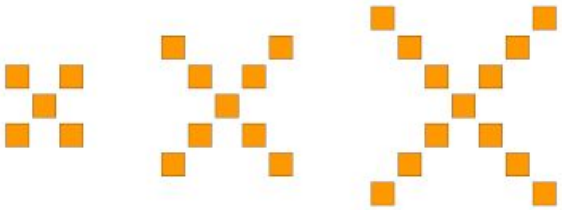
3. **Rekenrek:**
   - Slide the beads on the rekenrek to represent a number
   - Ask students what number is represented on the rekenrek



Toolkit Strategy

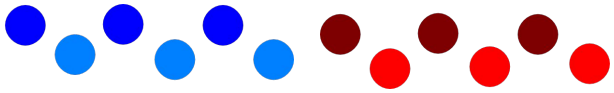
Number Sense: Visual Routines (cont.)

4. **Visual Patterns:**
   - Access an image on http://www.visualpatterns.org/
   - Display a visual pattern to students
   - Ask students to explain the pattern they see represented in the image and how the pattern is changing

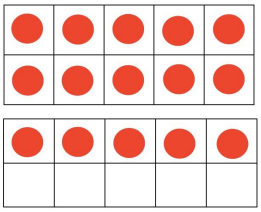


Additional Resources (Variations):

1. **Dot Cards:** Dots on dot cards can be the same color or shading, different shadings, or scattered.



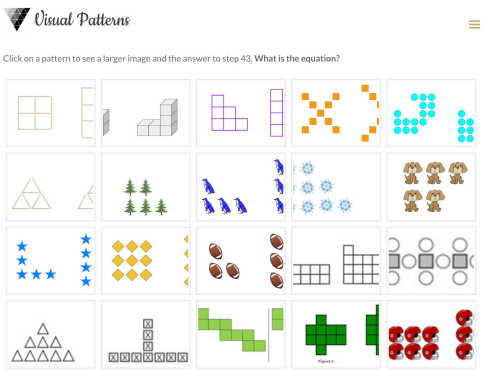
2. **Ten Frames:**
   - Combinations of 10
   - Amounts in the teens
   - Part-part-whole with 20 being the whole amount



3. **Rekenreks:** Use 20-100 bead rekenrek for higher numbers

4. Toy Theater

5. Visual Patterns



Distance Learning Tools: Jamboard, Google Slides, Nearpod, Pear Deck

Number Sense: Counting Routines

Toolkit Strategy

Number Sense: Counting Routines

SMP 4: Model with mathematics.
SMP 7: Look for and make use of structure.
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: TK - 12

Purpose:

- Understand place value and the number system

Description:

- Teachers engage students in routines that help build number sense, numerical literacy, and understanding of numbers.

When:

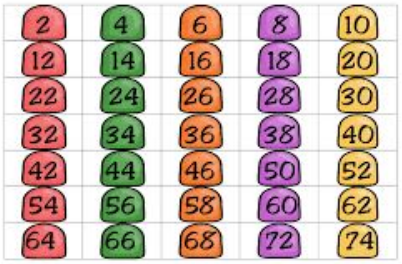
- Counting Routines can be used at any time during a math lesson, but generally are done at the beginning of the lesson.

Why:

- Provide daily number sense experiences
- Include discussion about numbers and their relationships
- Build on students' existing number sense
- Encourage students to play with numbers and enrich their mathematical thinking

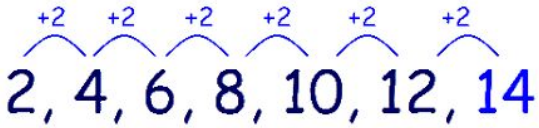
How:

1. **Count Around the Circle:**
   - Select a sequence to count by (twos, tives, tens, etc.) and a beginning number (for example, count by tens starting at twenty-six).
   - Go around the circle (or the room) as each student says a number counting on to the sequence (for example, the first person says "twenty-six", the next person says "thirty-six", and so on).



2. **Choral Counting:**
   - Select a sequence to count by and a beginning number (for example, count by twos starting at thirty-eight).
   - Class counts the number sequence aloud together ("thirty-eight", "fourty", and so on).

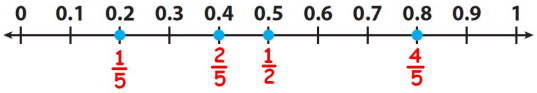
3. **Start and Stop Counting:**
   - Select a sequence to count by, a starting number, and a stopping number (for example, count by tens starting at fifty-six and stopping at one hundred thirty-six).
   - Class counts the number sequence aloud together ("fifty-six", "sixty-six", and so on until one hundred thirty-six).



Toolkit Strategy

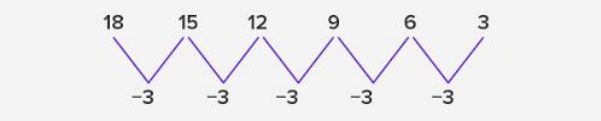
Number Sense: Counting Routines (cont.)

4. **Organic Number Line:**
   - Choose a section of a number line to magnify (for example from 0 to 1)
   - With students, identify additional numbers that can be found on the number line section (for example, between 0 and 1 find ¼, ½, ¾, 0.5, etc.).
   - Reinforce with students that each of the numbers can be represented in different ways and there are numbers that are equivalent.

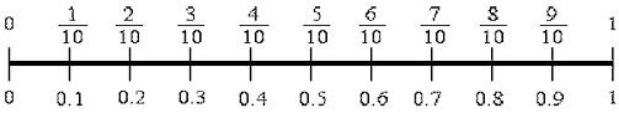


Additional Resources (Variations):

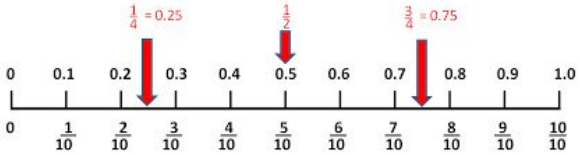
1. **Counting around the circle:**
   - Count by ones, tens, fives, twos, threes, etc., starting at zero.
   - Count by ones, tens, fives, twos, threes, etc., starting at various points.
   - Count by tens, starting from 320 (320, 330, 340, 350, 360, 370 . . .).
   - Count by tens, starting from 53 (53, 63, 73, 83, 93, 103, 113. . .).
   - Count backward by ones, tens, fives, and twos, starting at various points.
   - Count backward by tens, starting from 110 (110, 100, 90, 80 . . .).
   - Count backward by tens, starting from 322 (322, 312,302, 292 . . .).
   - Count by halves (0, ½, 1, 1½, 2, 2½, 3 . . .), starting at zero or at various points (16, 16½, 17, 17½, 18, 18½, 17, 18 . . .).
   - Count by fourths, eighths, thirds, or sixths, starting at zero or at various points.
   - Count by wholes, starting at a fractional number.
   - Count by hundreds, thousands, or millions, starting at zero or at various points



2. **Start and Stop Counting:**
   - Counting on and then back
   - Counting with decimals and fractions
   - Counting with odd/even patterns



3. **Organic Number Line:** Ask student questions about equivalency, doubling, halving, etc. based on the section of the number line you are focusing on



Distance Learning Tools: Jamboard, Google Slides, Nearpod, Pear Deck

Number Sense: Playing with Quantities

Toolkit Strategy

Number Sense: Playing with Quantities

SMP 4: Model with mathematics.
SMP 7: Look for and make use of structure.
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: TK - 12

Purpose:

- Make sense of numbers and relationships
- Play with quantities, breaking them apart and putting them back together
- Think about how numbers are compose and how the base ten place-value system works

Description:

- Teachers engage students in routines that help build number sense, numerical literacy, and understanding of numbers.

When:

- Playing with Quantities can be used at any time during a math lesson, but generally are done at the beginning of the lesson.

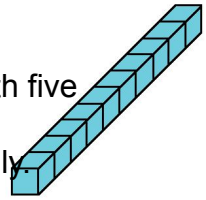
Why:

- Provide daily number sense experiences
- Include discussion about numbers and their relationships
- Build on students' existing number sense
- Encourage students to play with numbers and enrich their mathematical thinking
- Help students make connections to big ideas in mathematics

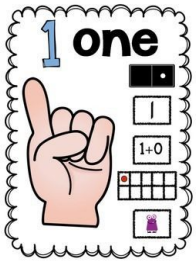
How:

- Teacher creates a number using various models (ten wands, drawing, story, and writing).
- Students think of strategies for composing and decomposing the number.
- Questions are asked to students to help them think flexibly and see relationships between the various place values.

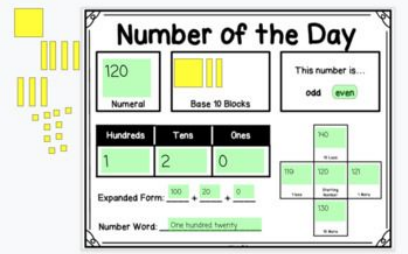
1. **Ten Wand:** Create a ten wand with ten Unifix cubes (or other linking cubes) with five of one color and five of a different color. The ten wand breaks in two pieces at various places (decomposing the ten) to help students see combinations visually.



2. **Ways to Make a Number:** Students write as many ways as they can think of to "make" a selected number. They might use visuals of the quantity, equations, models, etc.



3. **Today's Number:** Teacher chooses a number, such as ten, to be Today's Number and asks various questions about the number, such as: "When is ten big? When is ten small?"

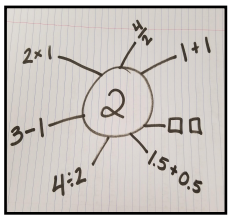


Toolkit Strategy

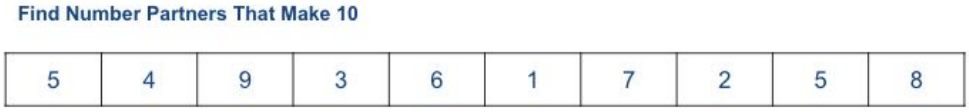
Number Sense: Playing with Quantities Routines (cont.)

4. **Mental Math:**
   - Teacher presents an equation or story problem and asks students to solve it mentally in their heads (without paper and pen or manipulatives).
   - Students are then asked to verbalize the strategies they used mentally.

5. **Number Web:** Give students a quantity and ask them to express the quantity in as many ways as possible (for example, 8 might be represented as 8+0, 4+4, 100-92, 64/8, 8 ones, 1 x 8, etc).

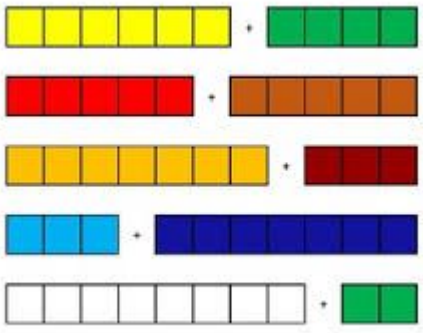


6. **Number Partners:** Give students ten numbers and ask them to find partners, or number pairs, that are equal to a specific number (for example, find number partners that are equal to 100).



Additional Resources (Variations):
Credit: "Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3" by Jessica F Shumway

1. **Ten Wand:**
   - Combinations of Ten
   - Part-Part Whole
   - Commutative Property
   - Using 5 and 10 Structure ------→



2. **Ways to Make a Number:**
   - Routine can be open ended
   - Routine can have constraints

3. **Today's Number**
   - Ask various questions for that day's number

4. **Mental Math routines for playing with quantities**
   - Decomposing numbers
   - Compensation
   - Making Tens
   - Counting On
   - Counting Back
   - Number Lines

Distance Learning Tools: Jamboard, Google Slides, Nearpod, Pear Deck

Number Sense: Calendar and Data Routines

Toolkit Strategy

Number Sense: Calendar and Data Routines

SMP 2: Reason abstractly and quantitatively.
SMP 7: Look for and make use of structure.
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: TK - 12

Purpose:

- Provide authentic times to use number sense
- Discuss authentic data

Description:

- Teachers engage students in routines that help build number sense, numerical literacy, and understanding of numbers.

When:

- Calendar and Data Routines can be used at any time during a math lesson, but generally are done at the beginning of the lesson.

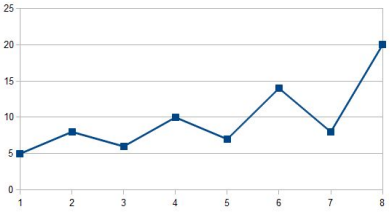
Why:

- Provide daily number sense experiences
- Include discussion about numbers and their relationships
- Build on students' existing number sense
- Encourage students to play with numbers and enrich their mathematical thinking

How:

1. **Calendar:**
   - As a class, write in the important days throughout the school year (birthdays, field trips, etc).
   - Ask questions and have students analyze days, dates,
   - occurrence of dates, difference of dates of events, etc.

2. **Collecting Data Over a Long Period of Time:**
   - With students, collect data (temperature, weather, etc.) over time and record the data on a graph in the classroom.
   - With students, discuss patterns and trends they notice in the data collected.



3. **Counting the Days in School:**
   - Create a number line beginning on the first day of school.
   - Every school day add the next number to the number line.
   - Throughout the year discuss and highlight various numbers or quantities (even numbers, groups of tens, etc.)

Toolkit Strategy

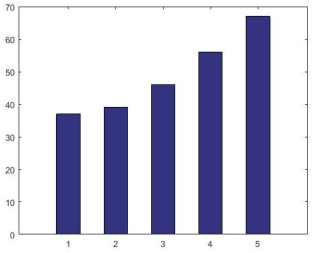
Number Sense: Calendar and Data Routines (cont.)

Additional Resources (Variations):
Credit: "Information from Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3" by Jessica F Shumway

1. **Calendar:**

- **Ask a variety of questions, such as:**
  - "Find the ____"
  - "Find the number that represents this amount." (show dot card)
  - "Find the number one less/one more than ___."
  - "Find a number greater than/less than/close to ____"
  - "What patterns do you notice? How does that help us?"
- **Use a twelve-month calendar to extend the questions.**

2. **Collecting Data Over a Long Period of Time:**
   - Use a variety of graph types (line graph, bar graph, etc.)
   - Summarize Data with Mode, Average, and Range
   - Bring in discussion of science concepts to think more deeply about why temperature changes
   - Assign Data Collection Jobs: Data Assistant, Weather Monitor, Daytime Tracker, Moon watcher, Calendar Keeper, Data Supervisor



3. **Counting the Days in School:**
   - Use different models: linear, array (hundreds chart), manipulatives (counting cubes in sticks of 10), etc.
   - For older students, use the calendar in conjunction with the days in school (consider weekends and days not in school)

Distance Learning Tools: Padlet, Jamboard, Google Slides, Nearpod, Pear Deck

“3 Reads as a routine, has been very helpful for me to differentiate for my language learners. They are able to focus on the story around the word problems and then bring attention to the numbers.”

Middle School Teacher

Toolkit Strategy:

Three Reads

Toolkit Strategy
Three Reads

SMP 1: Make sense of problems and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 6: Attend to precision.

Grade Span: 3 - 12

Purpose:
- Ensure students know what they are being asked to do in the mathematical context.
- Create opportunities for students to reflect on the ways mathematical questions are presented.

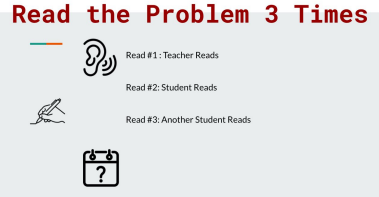
Description:
- A close read equivalent for a mathematical problem.
- Phases in the depth of word problems by layering the focus each time it is read.
- Students practice a process and structure to tackle word problems annotation.
- Reflective process to establish that the answer makes sense within the context.

When:
- During an application or word problem.

Why:
- Equip students with tools used to negotiate meaning (Kelemanik, Lucenta & Creighton, 2016).

How:
- The word problem is read three times (preferably by a different person each time):
  - First Read: to gain understanding of the context.
  - Second Read: to gain understanding of the mathematical idea.
  - Third Read: to plan for solving the problem.

Examples:



Credit: Illustrative Mathematics

Additional Resources:
- Teacher Guide
  - To help prepare and plan the 3 Reads with sentence frames
- Padlet Template
  - With sentence frames for 3 Reads (Password is ‘SVEF’)
  - Students can post and also have a chance to look at others’ responses and give peer feedback.
- Jamboard Template
  - Can be used to elicit responses in a virtual learning environment.
  - A text box with the word problem can be inserted in each frame.

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

“I love using ‘Routines for Reasoning’ with my students! The routines allow my students to focus on their thinking as they engage with the mathematics. ”

Elementary School Teacher 2021

Toolkit Strategy:

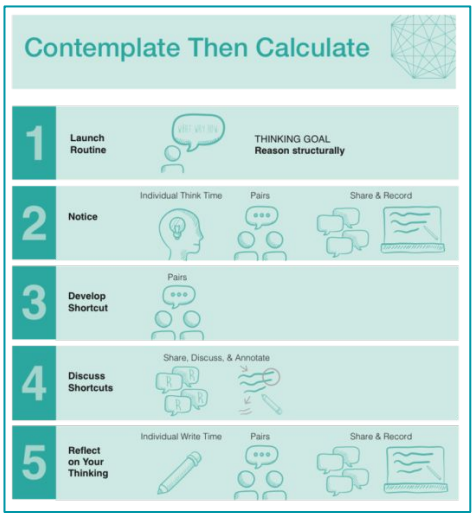
Routines for Reasoning

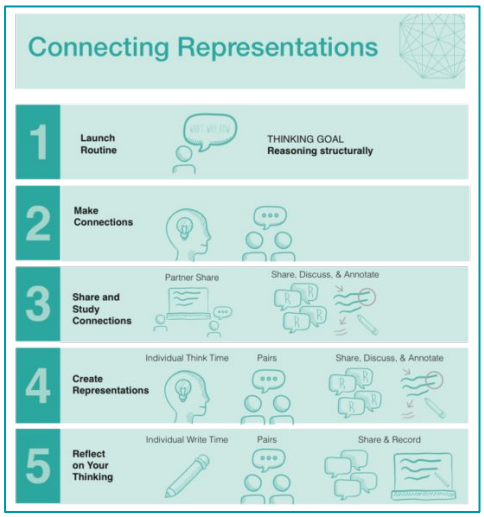
FOSTERING MATH PRACTICES

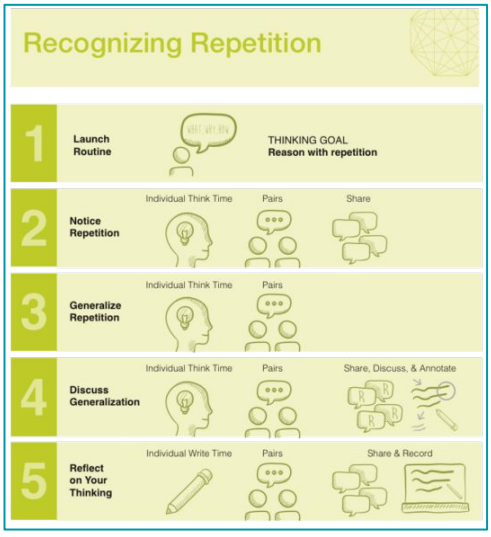
Toolkit Strategy
Routines for Reasoning

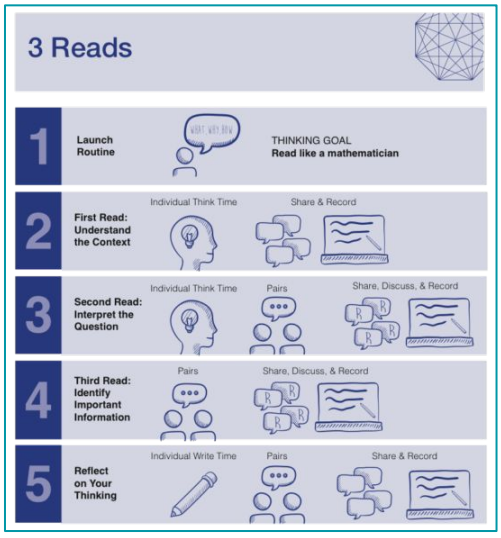
Instructional Routines are specific and repeatable designs for learning that support both the teacher and students in the classroom.

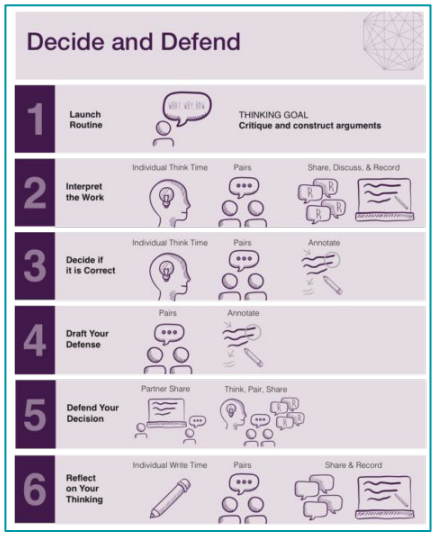
Each of the routines provide a familiar, accessible structure that supports repeated use until the steps to follow, thinking skills to employ, and questions to ask become automatic — enabling all students to engage more fully in learning opportunities while building crucial mathematical thinking habits.











Toolkit Strategy
Routine for Reasoning: Contemplate then Calculate

SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 6: Attend to precision.
SMP 7: Look for and make use of structure.

Grade Span: TK - 12

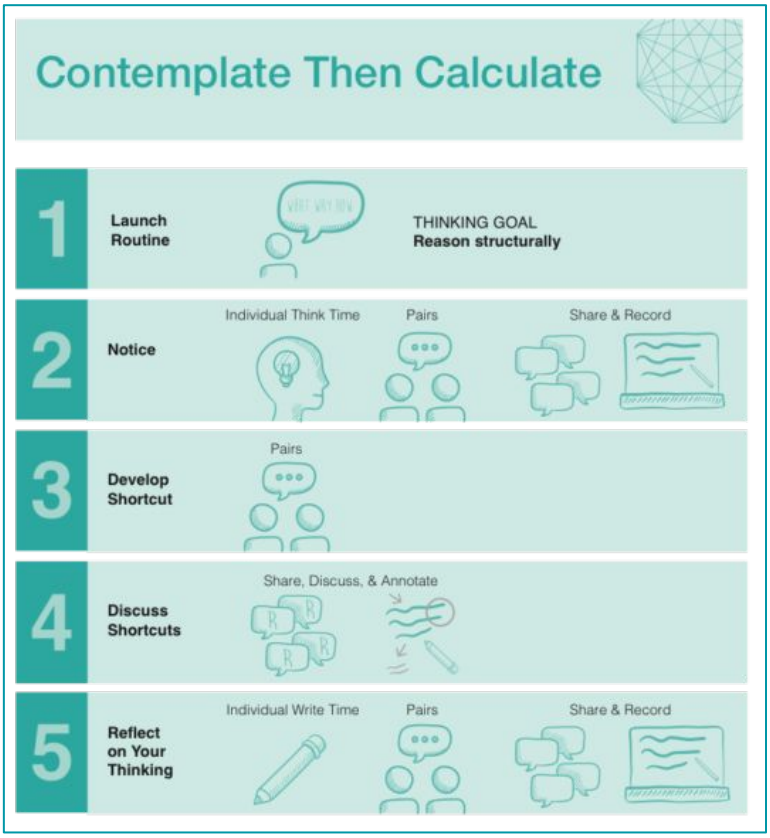
Purpose:
- Contemplate then Calculate is an instructional routine designed to shift attention away from mindless calculations and toward necessary structural interpretations of mathematics.

Description:
- Develops students' capacity to attend to mathematical structure when problem solving by using properties, rules of operations and relationships to uncover mathematical form and structure, and applying them to find calculation shortcuts or generalize results.

When:
- When solving an equation, graph, or pattern students make sense of a mathematical object (a visual pattern, arithmetic operation, equation, or graph) and look for what's structurally significant about it.

Why:
- Facilitate meaningful mathematical discourse
- Establish mathematics goals to focus learning
- Pose purposeful questions
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Elicit and use evidence of student thinking

How:

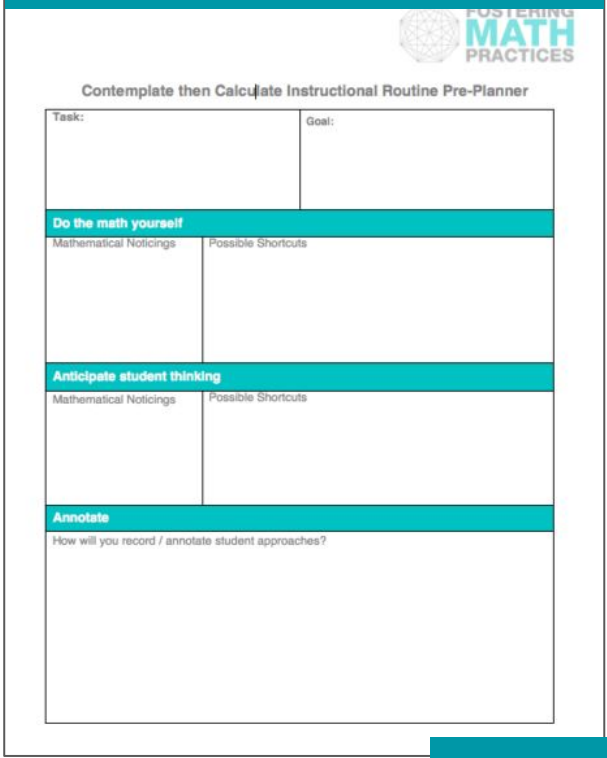


Toolkit Strategy

Routine for Reasoning: Contemplate then Calculate (cont.)

Additional Resources:

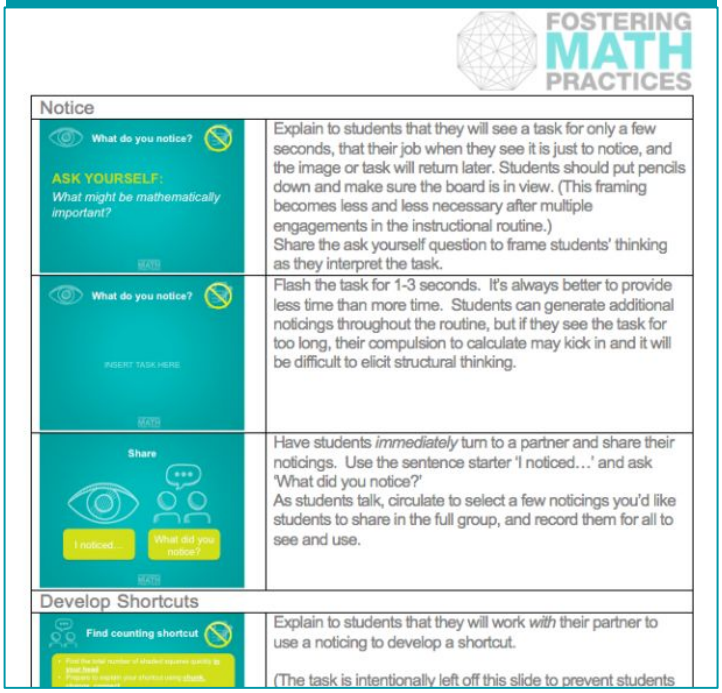
Word doc: Classroom Planner



Contemplate then Calculate Instructional Routine Pre-Planner

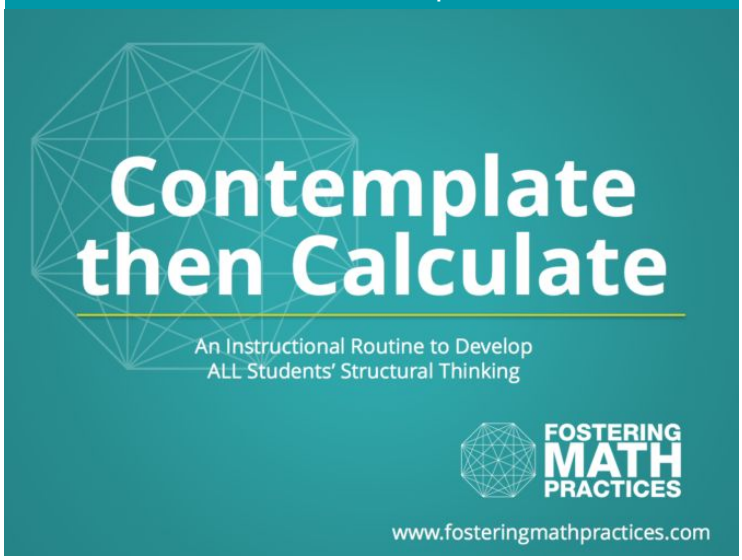
Task:	Goal:
Do the math yourself	
Mathematical Noticings	Possible Shortcuts
Anticipate student thinking	
Mathematical Noticings	Possible Shortcuts
Annotate	
How will you record / annotate student approaches?	

Contemplate then Calculate Overview



Notice	
What do you notice? ASK YOURSELF: *What might be mathematically important?*	Explain to students that they will see a task for only a few seconds, that their job when they see it is just to notice, and the image or task will return later. Students should put pencils down and make sure the board is in view. (This framing becomes less and less necessary after multiple engagements in the instructional routine.) Share the ask yourself question to frame students' thinking as they interpret the task.
What do you notice? INSERT TASK HERE	Flash the task for 1-3 seconds. It's always better to provide less time than more time. Students can generate additional noticings throughout the routine, but if they see the task for too long, their compulsion to calculate may kick in and it will be difficult to elicit structural thinking.
Share: I noticed... / What did you notice?	Have students *immediately* turn to a partner and share their noticings. Use the sentence starter 'I noticed…' and ask 'What did you notice?' As students talk, circulate to select a few noticings you'd like students to share in the full group, and record them for all to see and use.
Develop Shortcuts	
Find counting shortcut	Explain to students that they will work *with* their partner to use a noticing to develop a shortcut. (The task is intentionally left off this slide to prevent students

PowerPoint Slides: Contemplate then Calculate



Credit: FosteringMathPractices.com

Distance Learning Tools: Padlet, Jamboard, Google Slides, Nearpod, Pear Deck

Toolkit Strategy
Routine for Reasoning: Capturing Quantities

SMP 2: Reason abstractly and quantitatively.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- Focus attention on important quantities and relationships in problem situations.

Description:
- Look for and represent quantities and relationships represented in a problem or task by asking what they can count or measure and the relationships present.
- Students represent the important quantities and relationships in a diagram that they discuss with a partner or partners.

When:
- Solving word problems and diagrams and need to identify quantities and relationships

Why:
- To find quantities and relationships in word problems and diagrams

How:

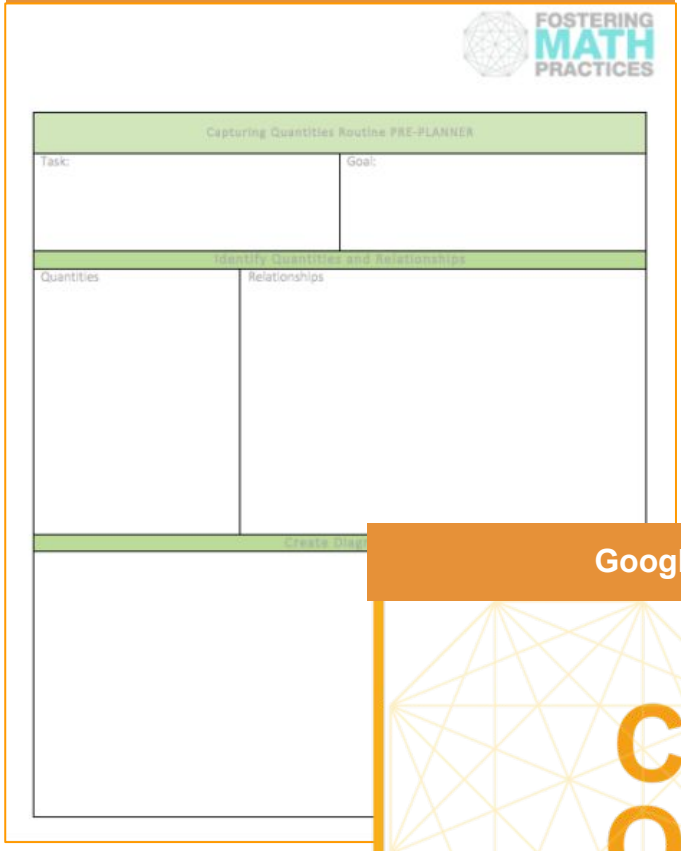
Credit: FosteringMathPractices.com

Toolkit Strategy

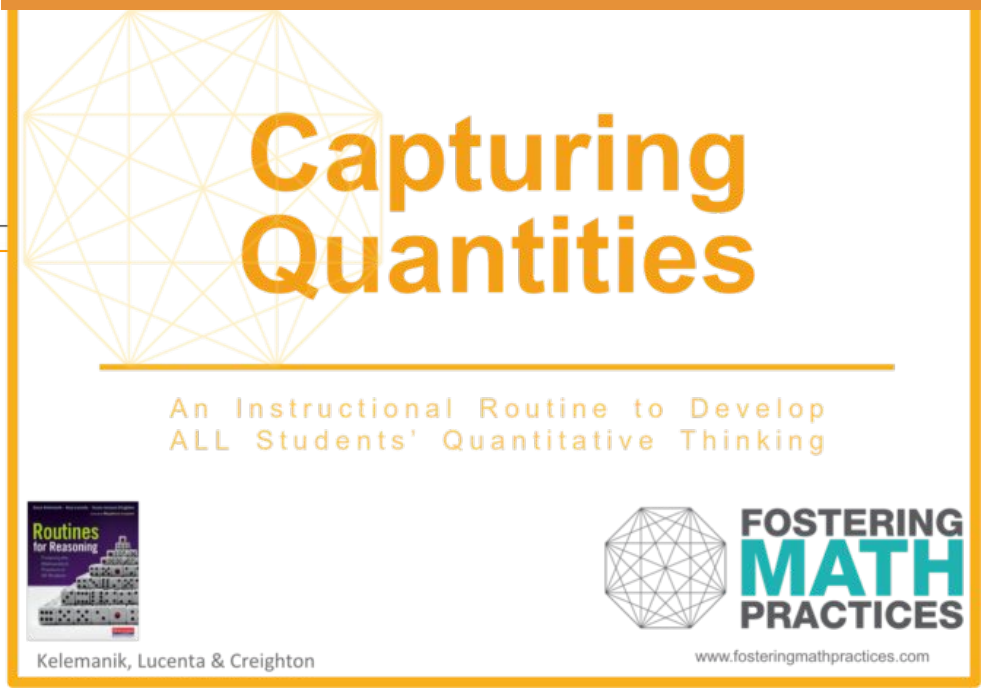
Routine for Reasoning: Capturing Quantities (cont.)

Additional Resources:

Word doc: Capturing Planner



Google Slides: Capturing Quantities



Credit: FosteringMathPractices.com

Distance Learning Tools: Padlet, Jamboard, Google Slides, Nearpod, Pear Deck

Toolkit Strategy
Routine for Reasoning: Connecting Representations

SMP 7: Look for and make use of structure.
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: TK - 12

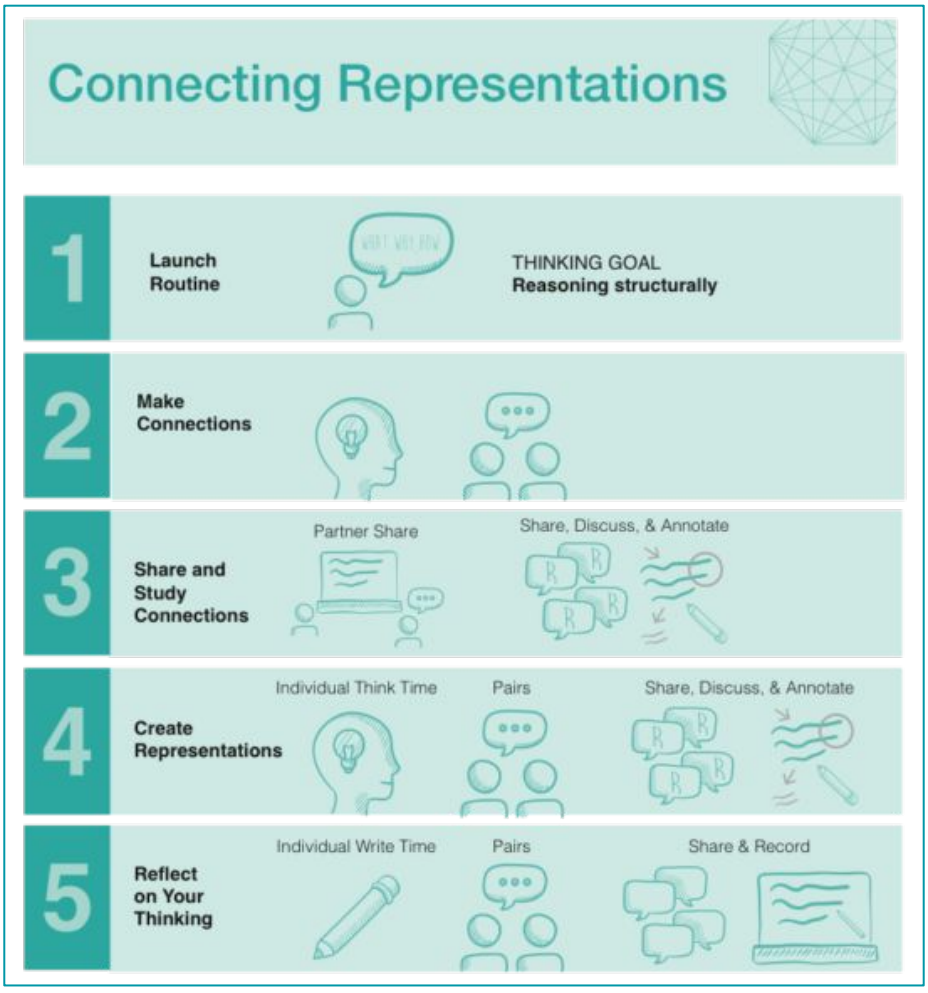
Purpose:
- Think structurally as students connect two representations by connecting the underlying mathematics.

Description:
- Use mathematical structure to match two different representations by matching visuals to expressions by chunking and connecting to math they know.

When:
- Need to match visuals to expressions and connect to the mathematics students know

Why:
- To use mathematical structure to match two different representations

How:



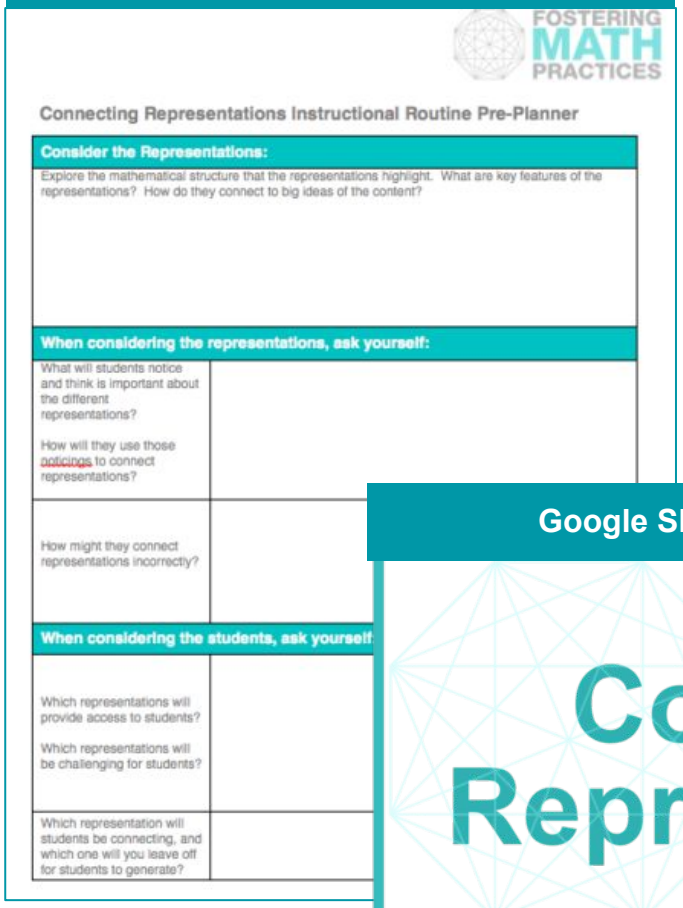
Credit: FosteringMathPractices.com

Toolkit Strategy

Routine for Reasoning: Connecting Representations (cont.)

Additional Resources:

Word doc: Connecting Representations Planner



FOSTERING MATH PRACTICES

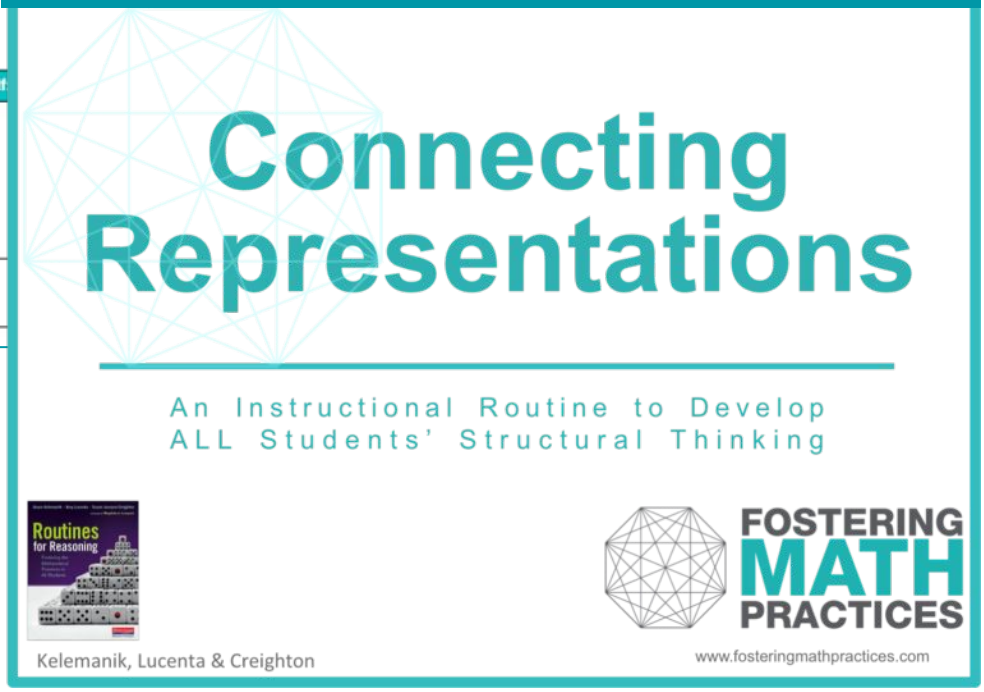
Connecting Representations Instructional Routine Pre-Planner

Consider the Representations:
Explore the mathematical structure that the representations highlight. What are key features of the representations? How do they connect to big ideas of the content?

When considering the representations, ask yourself:	
What will students notice and think is important about the different representations? How will they use those noticings to connect representations?	
How might they connect representations incorrectly?	

When considering the students, ask yourself:	
Which representations will provide access to students? Which representations will be challenging for students?	
Which representation will students be connecting, and which one will you leave off for students to generate?	

Google Slides: Connecting Representations



Credit: FosteringMathPractices.com

Distance Learning Tools: Padlet, Jamboard, Google Slides, Nearpod, Pear Deck

Toolkit Strategy
Routine for Reasoning: Recognizing Repetition

SMP 7: Look for and make use of structure.
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: TK - 12

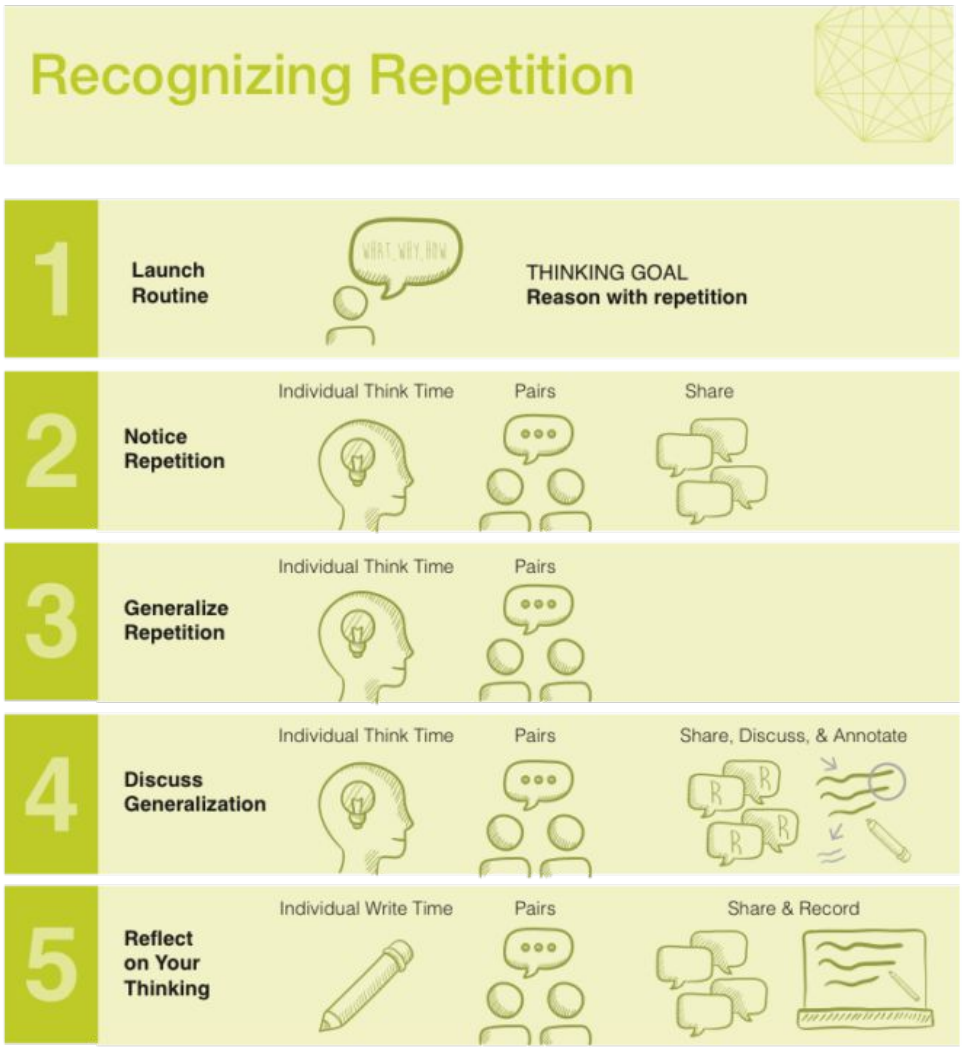
Purpose:
- Recognize repetition in order to make abstract generalizations.

Description:
- Leverage repeated reasoning to make abstract generalizations

When:
- When students are counting, calculating, and constructing and need to generalize a repetition

Why:
- To look for regularity in counting, calculating, and constructing processes

How:



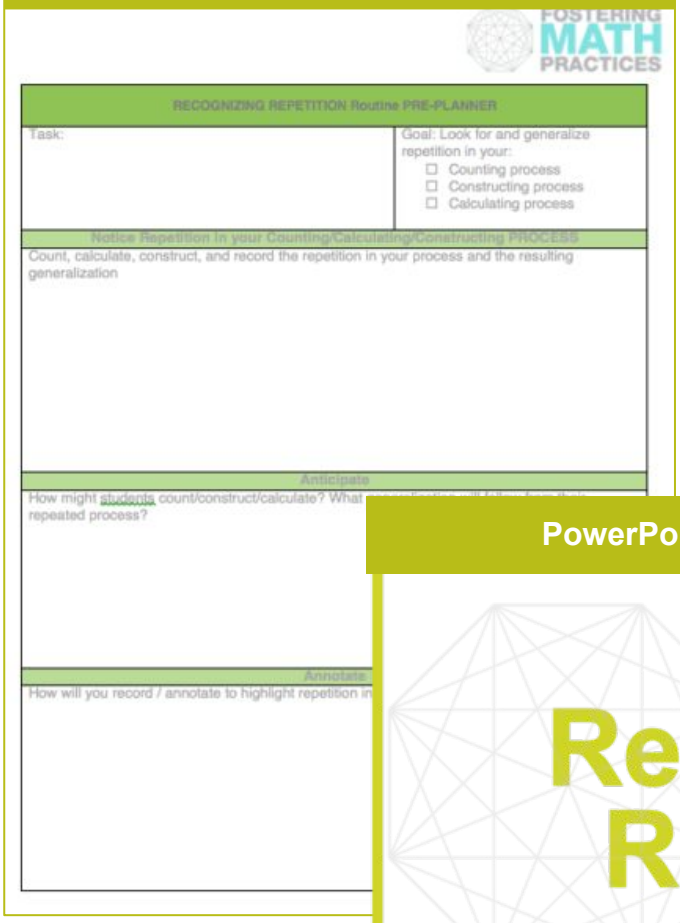
Credit: FosteringMathPractices.com

Toolkit Strategy

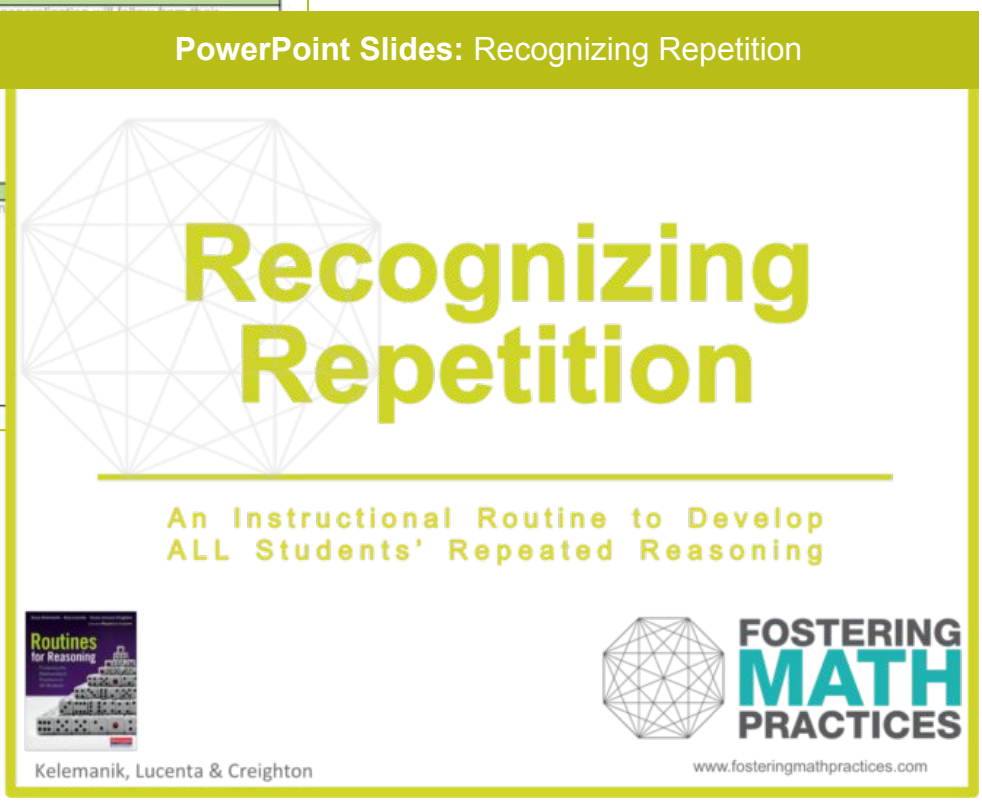
Routine for Reasoning: Recognizing Repetition (cont.)

Additional Resources:

Word doc: Recognizing Repetition Planner



PowerPoint Slides: Recognizing Repetition



Credit: FosteringMathPractices.com

Distance Learning Tools: Padlet, Jamboard, Google Slides, Nearpod, Pear Deck

Toolkit Strategy
Routine for Reasoning: Decide and Defend

SMP 3: Construct viable arguments and critique the reasoning of others
SMP 6: Attend to precision.

Grade Span: TK - 12

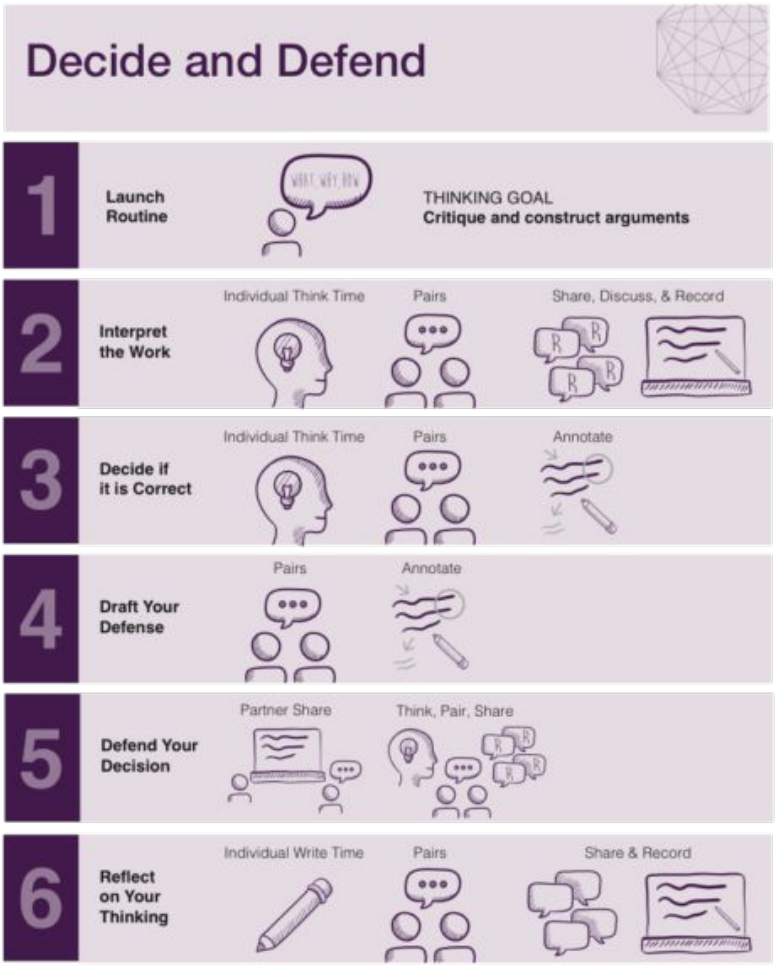
Purpose:
- Interpret math work to decide if they agree or disagree with the work and defend their decision.

Description:
- Make sense of another's line of mathematical reasoning, decide if they agree with that reasoning, then draft an argument defending their decision.

When:
- Students need to decide if they agree or disagree with mathematical work and defend their decision

Why:
- To support interpreting, deciding, and defending of mathematics reasoning

How:



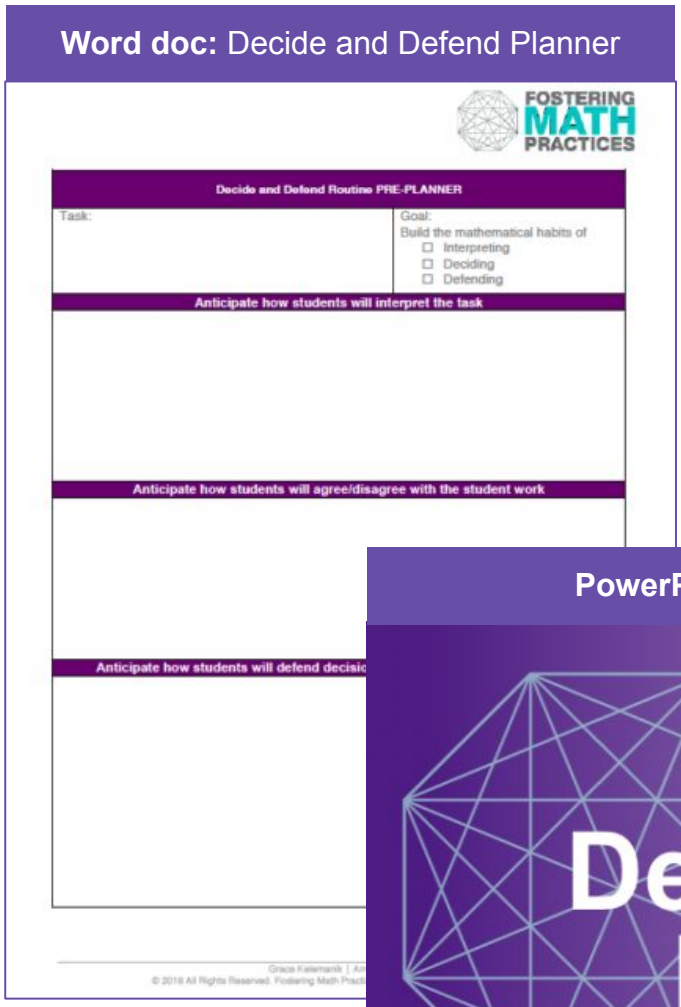
Credit: FosteringMathPractices.com

Toolkit Strategy

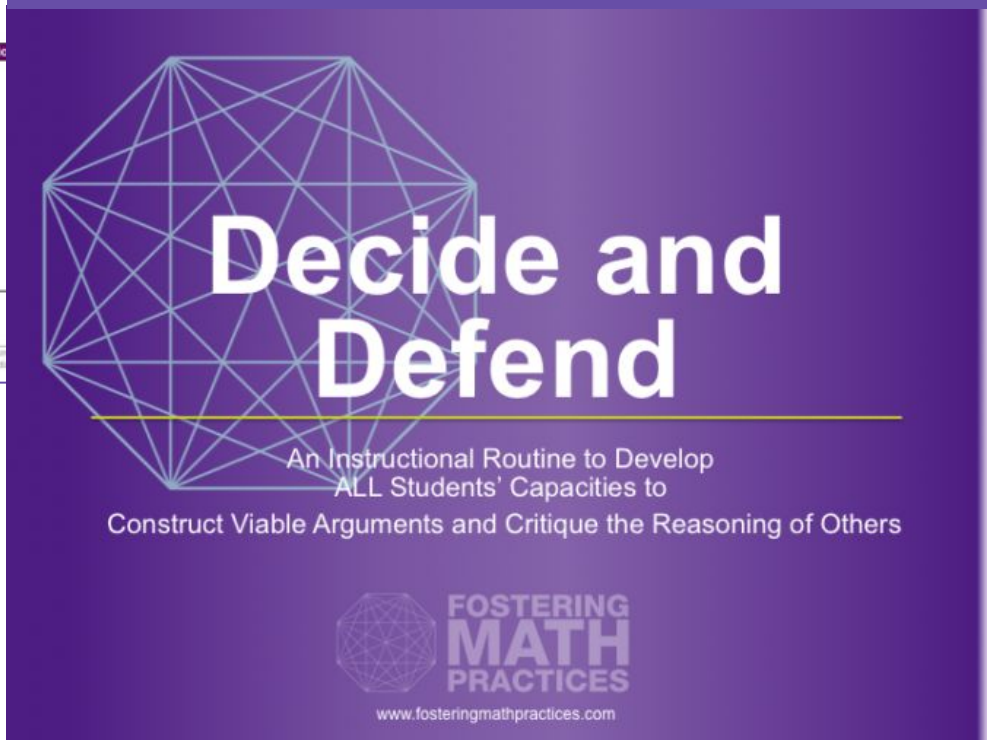
Routine for Reasoning: Decide and Defend (cont.)

Additional Resources:

Word doc: Decide and Defend Planner



PowerPoint Slides: Decide and Defend



Credit: FosteringMathPractices.com

Distance Learning Tools: Padlet, Jamboard, Google Slides, Nearpod, Pear Deck

Toolkit Strategy
Routine for Reasoning: Three Reads

SMP 1: Make sense of problems and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.

Grade Span: TK - 12

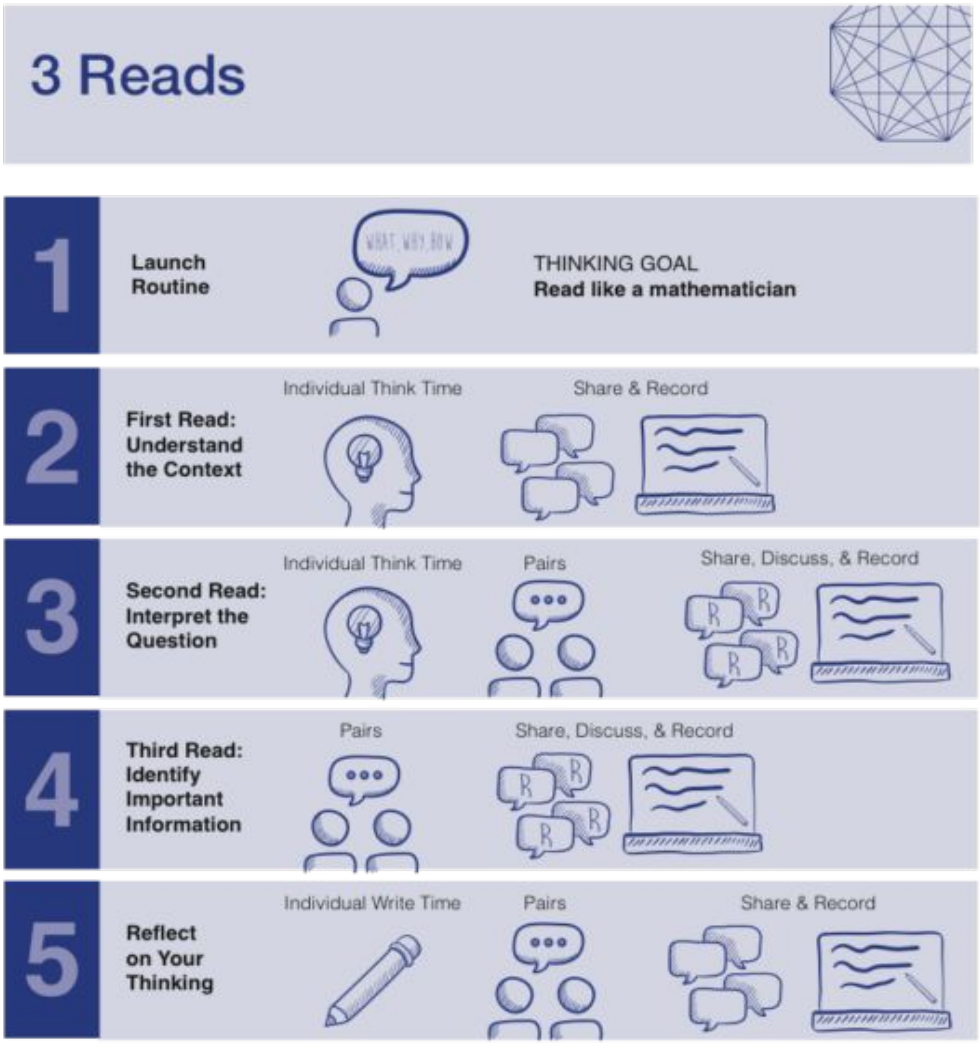
Purpose:
- "Read like a mathematician" and pay attention to quantities and relationships in a problem.

Description:
- Make sense of problems by deconstructing the process of reading mathematical situations.

When:
- Students are attempting to solve a word problem

Why:
- To make sense of a word problem and identify quantities and relationships in the problem

How:



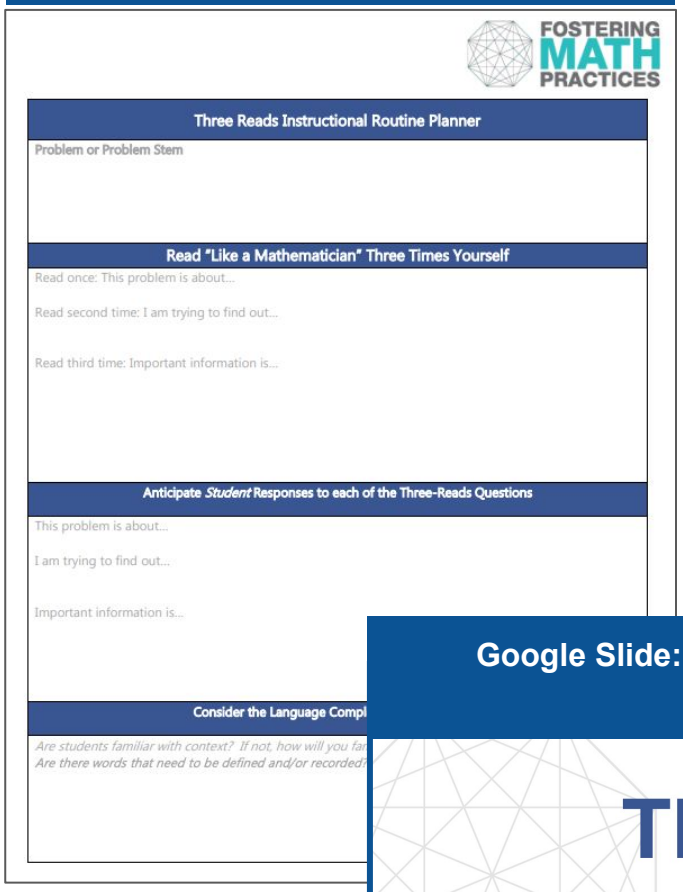
Credit: FosteringMathPractices.com

Toolkit Strategy

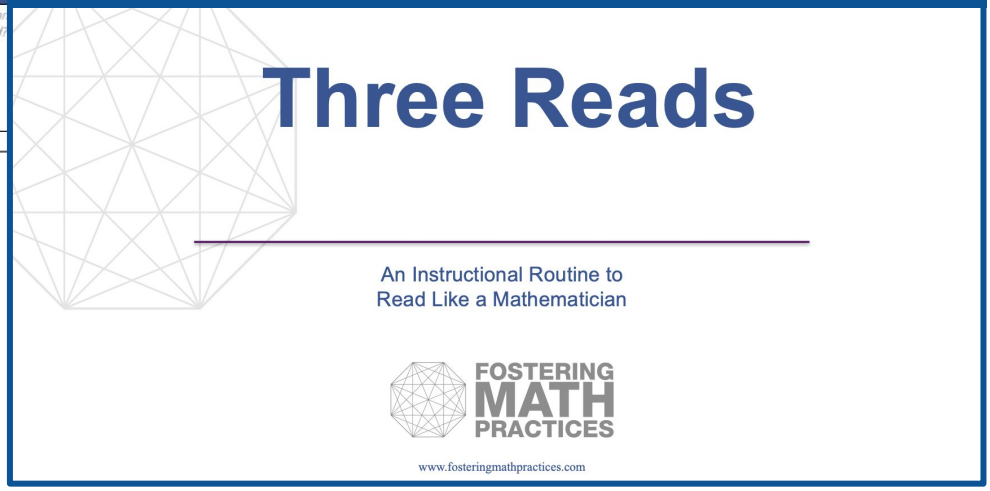
Routine for Reasoning: Three Reads (cont.)

Additional Resources:

Google Doc: Three Reads Instructional Routine Planner

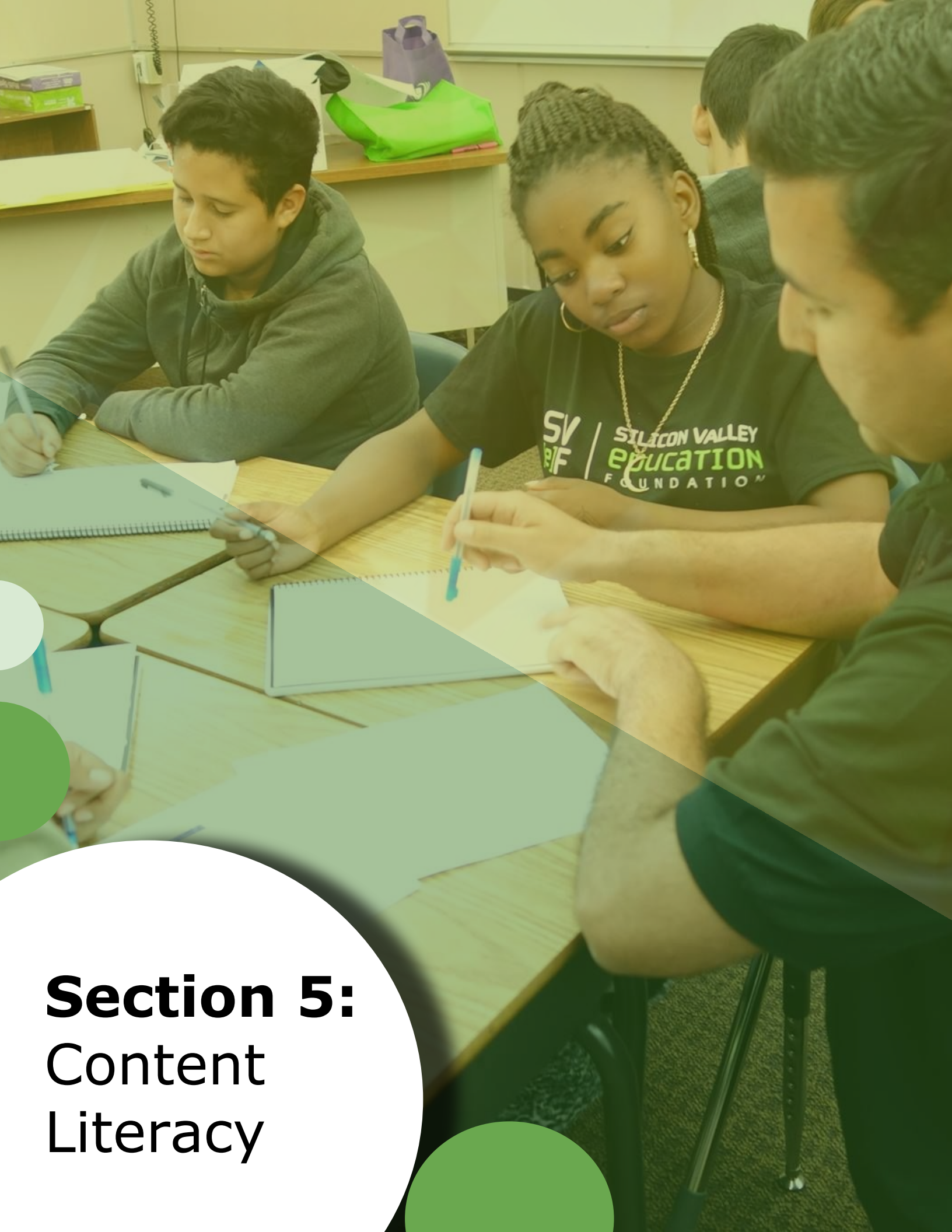


Google Slide: An Instructional Routine toRead Like a Mathematician



Credit: FosteringMathPractices.com

Distance Learning Tools: Padlet, Jamboard, Google Slides, Nearpod, Pear Deck



Section 5: Content Literacy

Section 5:

Content Literacy

Mathematical literacy is defined as:

> "*an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens*." (OECD, 2018, p. 67)

Language is essential in mathematics learning to enable students to develop their understanding of mathematics and to communicate their reasoning both verbally and in written form. Improving students' literacy in mathematics will help students to build connections between concepts, terminology, symbols, representations, and skills, contributing to the development of Mathematical literacy.

Mathematical Language Routines

Mathematical Language Routine (MLR) 1:
Stronger and Clearer Each Time

SMP 1: Make sense of problem and persevere in solving them.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014).

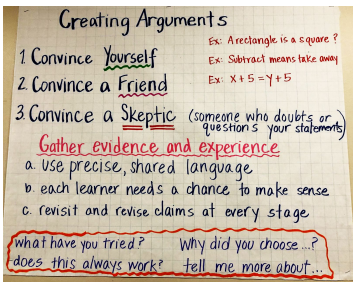
Description:
- Students think or write individually about a response.
- Through multiple pairing, students refine and clarify the response through conversation, to finally revise their original written response.

When:
- Students need to focus on details.
- Teachers need to see evidence of incorporating new ideas or language.

Why:
- To show evidence of refinement in precision, communication, expression, examples, and/or reasoning about mathematical concepts.
- To provide a purpose for student conversation and fortifies output.

How:
- Students think or write individually about a response, use a structured pairing strategy with multiple chances, to refine and clarify the response through conversation, and then finally revise their original written response.
- Students should be pressed for details, and encouraged to press each other for details. Subsequent drafts should show evidence of incorporating or addressing new ideas or language.
- Students show evidence of refinement in precision, communication, expression, examples, and/or reasoning about mathematical concepts.
- Convince Yourself, A Friend, And a Skeptic! (Video of Inside Mathematics - Cathy Humphrey's class where students justify their findings about diagonals of kites.)

Examples:



Credit: @mlangmyer

Additional Resources:
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

Mathematical Language Routine (MLR) 2:
Collect and Display

SMP 4: Model with mathematics.
SMP 7: Look for and make use of structure.

Grade Span: TK - 12

Purpose:
- Capture students' oral words and phrases into a stable, collective reference.

Description:
- The teacher listens for, and scribes, the language students use during partner, small group, or whole class discussions using written words, diagrams and pictures.
- Feedback for students increases sense-making while supporting meta- awareness of language.

When:
- Students need to refer throughout the unit to build on, or make connections with during future discussions.
- Bridging student language by updating and revising the display as student language changes, and new disciplinary language are made.

Why:
- Feedback for students increases sense-making while supporting meta-awareness of language.
- Stabilizes the fleeting language that students use in order for their own output to be used as a reference in developing their mathematical language.

How:
- During individual/ pair/ group work, circulate and listen to student(s) talk or watch them scribe their thoughts. Jot notes about common or important words and phrases observed.
- Scribe students' words and sketches on visual display to refer back to during whole class discussions throughout the unit.
- Refer back to these words, phrases/ diagrams by asking students to explain how they are useful, asking students to clarify their meaning, and asking students to reflect on which words and visuals help to communicate ideas more precisely.

Examples:

Gather and Show Student Discourse (Dieckmann, 2017)
- During pair/group work, circulate and listen to student talk during pair work or group work, and jot notes about common or important words and phrases, together with helpful sketches or diagrams. Scribe students' words and sketches on visual display to refer back to during whole class discussions throughout the unit. Refer back to these words, phrases, and diagrams by asking students to explain how they are useful, asking students to clarify their meaning, and asking students to reflect on which words and visuals help to communicate ideas more precisely.

Additional Resources:
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

Mathematical Language Routine (MLR) 3:
Critique, Correct and Clarify

SMP 1: Make sense of problem and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.

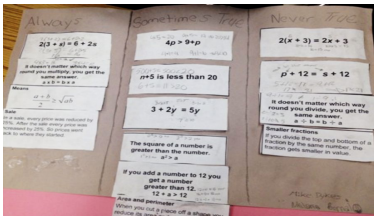
Description:
- This will prompt student reflection with an incorrect, incomplete, or ambiguous written argument or explanation, and for students to improve upon the written work by correcting errors and clarifying meaning.

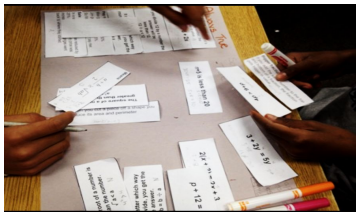
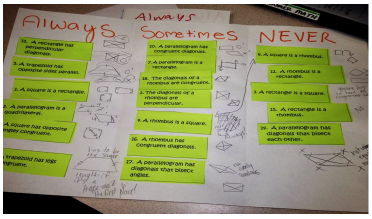
When:
- Modeling how to effectively and respectfully critique the work of others with meta-think-alouds and press for details when necessary.

Why:
- Fortifies output and engages students in meta-awareness.

How:
- Teacher can prompt students to reflect on an incorrect, incomplete, or ambiguous written argument or explanation, and students work to improve upon the written work by correcting errors and clarifying meaning.
- Teacher can present a partial/broken argument, explanation, or solution method, and ask for help in fixing it. Students identify the error(s), analyze the response in light of their own understanding of the problem, and can work both individually or in pairs to propose an improved response.
- Teacher can then have a whole class share out for improved response, and students refine their own draft response.
- Always- Sometimes- Never

Examples:



Credit: Fawn Nguyen (left and middle) and Lisa Bejarano (right)

Additional Resources:
- Always, Never, Sometime- Nrich Tasks (includes Teacher Resources)
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

Mathematical Language Routine (MLR) 4:
Information Gap

SMP 1: Make sense of problem and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- Create a need for students to communicate (Gibbons, 2002).

Description:
- Teachers model how to ask for and share information, clarification, justification, and elaboration.

When:
- Students need to orally (and/or visually) share ideas and information to bridge the gap and accomplish something that they cannot do alone.

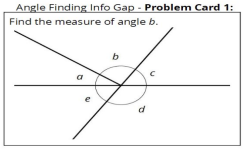
Why:
- Facilitates meaningful interactions among partners or team members, to put pieces of necessary information together to solve a problem or play a game.
- Cultivates conversation.

How:
- Students can play a guessing game or matching game in which they have a real reason to talk (e.g., students need to work together to develop a strategy to win a game; each student is provided with different information; one student has something in mind and other students use their understanding of a mathematical concept to guess what it is).
- In one version of this activity, Partner A has a problem on a card, and Partner B has the information needed to solve it on the "data card." Partner A identifies what is needed and asks for information that is provided on Partner B's data card. Partner B does not share information unless Partner A asks for it. Neither partner should read their cards to one another nor show their cards to their partners. As they work, they justify their responses using clear and connected language.

Examples:

Angle Finding Info Gap - **Problem Card 1:**

Find the measure of angle *b*.

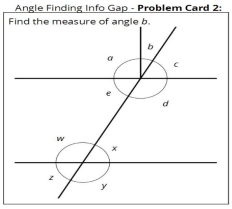


Angle Finding Info Gap - **Data Card 1:**

- Angles *c* and *e* are vertical angles.
- Angles *a* and *c* are complementary angles.
- The measure of angle *d* = 124°.
- The measure of angle *c* = 56°.

Angle Finding Info Gap - **Problem Card 2:**

Find the measure of angle *b*.



Angle Finding Info Gap - **Data Card 2:**

- The measure of angle *a* = 90°.
- The measure of angle *z* = 63°.
- Angles *c* and *z* are not vertical angles.
- Angles *d* and *x* are supplementary angles.

Credit: [*Illustrative Mathematics (Lesson 7.4.2)*]

Additional Resources:
- Education.com
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

Mathematical Language Routine (MLR) 5:
Co-craft Questions and Problems

SMP 1: Make sense of problem and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- Allow students to get inside of a context before feeling pressure to produce answers.
- Create space for students to produce the language of mathematical questions themselves.

Description:
- Students use conversation skills to generate, choose (argue for the best one), and improve questions, problems, and situations as well as develop meta-awareness of the language used in mathematical questions and problems.

When:
- Teachers need to push for clarity and re-voice oral responses as necessary.

Why:
- Provide opportunities for students to analyze how different mathematical forms can represent different situations.

How (Source: Illustrative Mathematics Blog)
- Teacher can present a situation/ problem/ mathematical representation.
- Students write down possible mathematical questions about the situation or solve their own problems before trading them with other pairs or write stories or situations that correspond to the representation.
    - In pairs, students compare their questions or solve other pairs' problems, check solutions and methods with the other pairs or explain how their partner's story show mathematical representation, and share as whole class.

Examples: Co-Craft Situations
1. **PRESENT REPRESENTATION:** Teacher presents a mathematical representation (graph, equation, function, table, etc.) with no labels.
2. **STUDENTS WRITE:** Students write stories or situations that correspond to the mathematical representation.
3. **PAIRS COMPARE:** Students explain how events in their partner's story or aspects of their partner's situation correspond to specific parts of the mathematical representation. They can ask clarifying questions or for more detail to do this.
4. **REVISE STORIES:** Students revise their stories adding details and clarification where needed.

Additional Resources:
- Illustrative Mathematics Blog
- Education.com
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

Mathematical Language Routine (MLR) 6:
Three Reads

SMP 1: Make sense of problem and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- Ensure students know what they are being asked to do in the mathematical context.
- Create opportunities for students to reflect on the ways mathematical questions are presented.

Description:
- A mathematical equivalent of what close read is for language.
- Phases in the depth of word problems by layering the focus each time when read.
- This deconstruction helps students practice a process and structure for tackling word problems by annotating notes for reading in mathematics while identifying a question within a word problem.

When:
- Deeper understanding of word problems and open ended questions are to be addressed.
- Chunking down complex language to clarify content for learners.

Why:
- Understand the mathematical inference of the words within, while keeping the engagement high.
- Reflect on their mathematical process and final answer to determine if the answer makes sense within the context.

How:
- Students are supported in reading a situation/problem three times, each time with a particular focus:
  - First Read: with the goal of comprehending the text (describe the situation without using numbers)
  - Second Read: with the goal of analyzing the language used to present the mathematical structure or idea.
  - Third Read: to brainstorm possible mathematical solution methods.
- Edutopia Video

Examples:

Credit: SFUSD

Additional Resources:
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)
- Teacher Guide
  - To help prepare and plan the 3 Reads with sentence frames
- Padlet Template
  - With sentence frames for 3 Reads (Password is 'SVEF')
  - Students can post and also have a chance to look at others' responses and give peer feedback.
- Jamboard Template
  - Can be used to elicit responses in a virtual learning environment.
  - A text box with the word problem can be inserted in each frame.

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

Mathematical Language Routine (MLR) 7:
Compare and Connect

SMP 4: Model with mathematics.
SMP 6: Attend to precision.
SMP 7: Look for and make use of structure.

Grade Span: TK - 12

Purpose:
- To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.

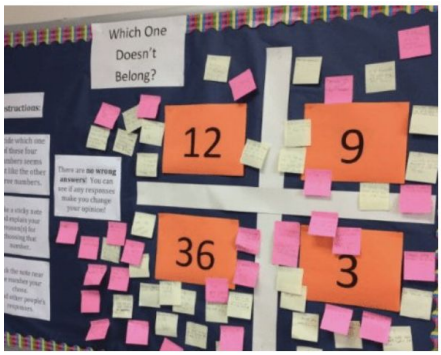
Description:
- Students reflect on and linguistically respond to these comparisons.
- Teachers model thinking out loud about these questions.

When:
- Exploring why or when one might do/say something a certain way, identifying and explaining correspondences between different mathematical representations or methods, wondering how an idea compares or connects to other ideas and/or language.

Why:
- Supports metacognitive and metalinguistic awareness.
- Supports mathematical conversation.

How:
- Teacher can asks students to show one strategy for a problem. Then they should focus on understanding another student's solution, and students will identify what is similar and what is different before sharing out with the class.
- Which One Doesn't Belong- Students are provided with sets of four numbers, equations, expressions, graphs, or images. They must decide how to group the sets so that three of the items fit within a category and one does not, and be prepared to justify which item did not fit.

Examples:



Credit: #mathmovement : Blog by Sara Van Der Werf

Additional Resources:
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)

Distance Learning Tools: Jamboard, Padlet, Google Slides, SeeSaw

Mathematical Language Routine (MLR) 8:
Discussion Supports

SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 4: Model with mathematics.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009).

Description:
- Can be combined and used together with any of the other routines.
- Include multi-modal strategies for helping students make sense of complex language, ideas, and classroom communication.

When:
- Invite and incentivize more student participation, conversation, and meta-awareness of language.

Why:
- Allows students to begin using these strategies themselves to prompt each other to engage more deeply in discussions.

How:
- Revoice student ideas to model mathematical language by restating a statement as a question to clarify, apply appropriate language, and involve more students.
- Ask for details in students' explanations by requesting to challenge an idea, elaborate on an idea, or give an example.
- Show central concepts by utilizing different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gesture, and talking about the context of what is happening.
- Practice phrases or words through choral response.
- Think aloud by talking through a mathematical concept while solving a problem or doing a task.
- Model detailing steps, describing and justifying reasoning, and questioning strategies.

Examples: Whole Class Discussion Supports
- Revoice student ideas to model mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
- Press for details in students' explanations by requesting for students to challenge an idea, elaborate on an idea, or give an example.
- Show central concepts multi-modally by utilizing different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gesture, and talking about the context of what is happening.
- Practice phrases or words through choral response.
- Think aloud by talking through thinking about a mathematical concept while solving a related problem or doing a task. Model detailing steps, describing and justifying reasoning, and questioning strategies.

Additional Resources:
- Understanding Language/Stanford Center for Assessment, Learning, and Equity (2017)

Distance Learning Tools: Jamboard, Padlet, Google Slides, Flipgrid, SeeSaw

“This is very useful for Newcomers and EL students. Repeat usage in sentence frames strengthens content literacy.”

Middle School Teacher

Toolkit Strategy:

Word Wall

Toolkit Strategy
Word Wall

SMP 4: Model with mathematics.
SMP 5: Use tools strategically.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- Provide a display of mathematical content words to assist in vocabulary development.

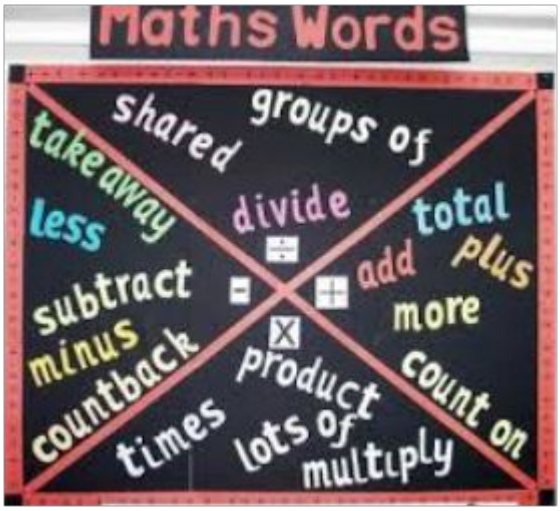
Description:
- Mathematics word wall is a display of content vocabulary words and key terms for students during a lesson or unit.

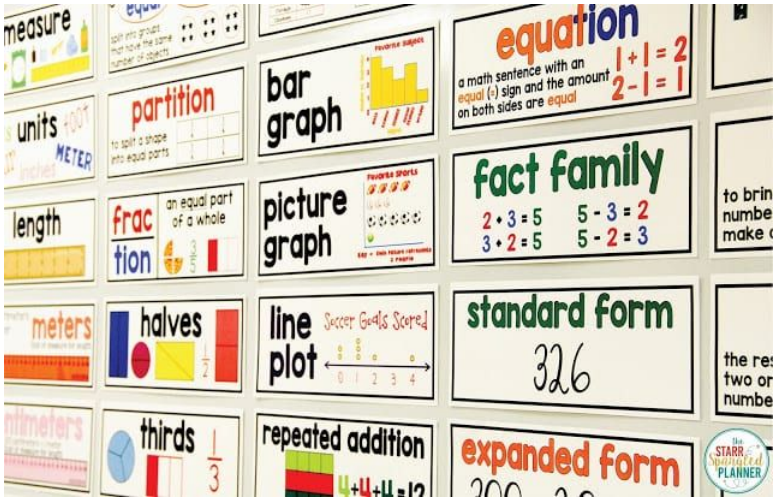
When:
- Designed as a tool for oral class discussion, reading and writing.
- Displayed on bulletin before and throughout the lesson or unit.

Why:
- Word walls play a key role in vocabulary and language development.
- Used as an instructional tool for educators and as a reference for English Language Learners.

How *(Source: Math Word Wall)*
- Introduce new words in context including diagrams or visuals
- Explain the meaning carefully and revisit the words frequently in a variety of examples.
- Have a display of the selected vocabulary words where students can see them and use them for math discussion and writing.
- Build the math wall by adding more vocabulary words.
- Encourage the use of the math word wall when working in math journals.

Examples:





Additional Resources:
- Math Word Wall Sample

Distance Learning Tools: Jamboard, Padlet, Google Slides

“Math Journals is a wonderful tool to encourage students to reflect and record their strategies, thought processes and perspectives on problem-solving or any mathematical concept. It creates an environment to allow students to be more engaged, open-minded, accountable and confident in math.”

Middle School Teacher

Toolkit Strategy:

Math Journal

Toolkit Strategy
Math Journal

SMP 4: Model with mathematics.
SMP 6: Attend to precision.

Grade Span: TK - 12

Purpose:
- To promote math literacy.
- Enable problem solving and use the language of math in everyday situations.
- Build a strong foundation using academic math vocabulary.

Description:
- Take notes
- Record new vocabulary
- Draw models
- Add examples
- Solve with and record different strategies
- Reference while solving a problem
- Reflect on learning

When:
- Throughout a math lesson as needed and/or appropriate

Why:
- Regular use of interactive math journal increases precision in math vocabulary
- Promotes meaningful student discourses and discussions
- Students have the opportunity to reflect on their strategies and assess their own learning
- Creates a documented record of student growth and progress

How:
- Students respond to math journal prompts as part of a “Math Notebook”.

- Possible Prompts for Math Journal:

    - **Prompts that assess attitudes or beliefs:**
        - Students write about their personal thoughts and feelings about Math.
            - Examples:
                - When it comes to math, I find it difficult to…
                - I love math because…
                - People who are good at math…
                - When I study for a math test, I….

    - **Prompts that assess learning:**
        - Students write about what they’ve learned and reflect on what they know (and don’t know).
            - Examples:
                - The most important thing I learned today is…
                - I could use today’s skill in my real life when I…
                - Today I used math when…
                - At the end of this unit, I want to be able to…
                - Some good test questions for this skill are….

Toolkit Strategy
Math Journal (cont.)

- **Prompts That Assess Process:**
  - Students explain how to solve problems or discuss a particular skill or strategy.
    - Examples:
      - Two ways to solve this problem are…
      - I knew my answer was right when…
      - Another strategy I could have used to solve this problem is…
      - If I missed a step in this problem
      - I could have…
      - The most important part of solving this problem is to remember….
- Share responses from journals, either with a partner, group, whole class, etc.
- Model and provide numerous opportunities for students to use their math journal.

Examples:

- **Lesson Plans** *(Credit: Angela Watson)*
  - **Model lesson script for introducing math journaling to grades K-1**
  - **Model lesson script for introducing math journaling to grades 2-4**

Additional Resources:

- Prompts-for-writing-in-mathematics

Distance Learning Tools: Jamboard, Google Slides, Pear Deck, Nearpod, Virtual Journal

“Sentence frames and sentence starters are very supportive and provide structures for my students to use in communicating their thinking in a more academic register.”

Elementary School Teacher

Toolkit Strategy:

Sentence Frames

Toolkit Strategy
Sentence Frames

SMP 1: Make sense of problems and persevere in solving them.
SMP 3: Construct viable arguments and critique the reasoning of others.

Grade Span: 1 - 12

Purpose:

- Mathematical discussions is the way students represent, think, talk, question, agree, and disagree in the classroom.
- Teachers use language frames to model language for their students in order to create language behaviors their students can use when working cooperatively, discussing their work, and writing.

Description:

- Phrases used to scaffold student oral language and writing in order to explain math processes and concepts.
- A sentence frame is a question or sentence with words removed to provide a language or writing support for students.
- Usually the sentence frame consists of a subject and a predicate.
- By using sentence frames in math, teachers can offer a method of scaffolding for students as they build and develop math skills.
- Sentence frames give students an opportunity to access the math concepts and to engage in the classroom conversation of math by answering questions.

When:

- Sentence frames are used to support students to express their thinking.

Why:

- Sentence frames give students a chance to use the new math vocabulary in a meaningful way.
- Sentence frames support students' ability to produce language at a higher language level then they are at.

How:

- Teacher models the use of the sentence frames, including any new math vocabulary presented in the lesson.
- Post the frame(s) in the classroom for students to refer to.
- Monitor student discussions and writing for their use of the sentence frames
- Add additional language frames as students expand their content vocabulary and language development.
- Students use the sentence frames to write about what they were learning and/or share their learning orally.

Toolkit Strategy
Sentence Frames (cont.)

Examples:

SAMPLE LANGUAGE FRAMES FOR MATHEMATICS

- In order to solve this problem, I need to know _____.
- This is a _____ problem because I see _____.
- I started with an estimate by _____.
- We used the problem-solving strategy _____ and our answer is _____ because _____.
- In order to _____, we follow these steps _____.
- I use the _____ operation because the question asked me to _____.
- Describe the process: First, I _____. (step/process) Then, I _____. (step/process) Next, I _____. Finally, I _____.
- My/our answer is _____. I/we think this answer is reasonable because _____.
- Another way to solve this would be _____.
- Can you explain how/why _____?
- If I change _____, my answer would be different because _____.
- I respectfully agree/disagree with _____ because _____.
- I can check my answer by _____.

Retrieved from the companion website for *Visible Learning for Mathematics, Grades K–12: What Works Best to Optimize Student Learning* by John Hattie, Douglas Fisher, Nancy Frey, Linda M. Gojak, Sara Delano Moore, and William Mellman. Thousand Oaks, CA: Corwin, www.corwin.com. Copyright © 2017 by Corwin. All rights reserved. Reproduction authorized only for the local school site or nonprofit organization that has purchased this book.

Figure 5.7

SENTENCE FRAMES THAT CAN BUILD METACOGNITIVE THINKING

Planning
I wonder why . . .
I already know something about this topic. It is . . .
I know the word _____, but I don't know what _____ and _____ mean.
I see lots of graphics and charts. I'll need to use those to help me understand what I'm reading.
I think/believe/assert that . .
Monitoring
This wasn't what I expected. I expected _____ because _____.
What can I write or draw that might help me remember and understand?
Based on . . . I infer . . .
My idea/answer is similar to . . . because . . .
My idea/answer is different from . . . because . . .
I discovered from . . . that . . .
I found out from . . . that . . .
A question I have is . . . because . . .
Evaluating
How well did I understand the task or problem?
What strategies worked well for me?
What strategies did not work for me?
What should I do next time?
Do I need some help for next time?

Retrieved from the companion website for *Visible Learning for Mathematics, Grades K–12: What Works Best to Optimize Student Learning* by John Hattie, Douglas Fisher, Nancy Frey, Linda M. Gojak, Sara Delano Moore, and William Mellman. Thousand Oaks, CA: Corwin, www.corwin.com. Copyright © 2017 by Corwin. All rights reserved. Reproduction authorized only for the local school site or nonprofit organization that has purchased this book.

Figure 4.7

Additional Resources:

- Teacher Toolkit: Sentence Stems (Elementary School)
- Teacher Toolkit: Sentence Stems (High School)

Distance Learning Tools: Jamboard, Google Slides, Pear Deck, Nearpod, Padlet, Virtual Journal

"Mathematical tasks have brought in more conversations and have given my learners the opportunity to access rich tasks with multiple avenues to represent."

Middle School Teacher

Toolkit Strategy:

Math Tasks

Toolkit Strategy
Math Tasks

SMP 1: Make sense of problems and persevere in solving them.
SMP 3: Construct viable arguments and critique the reasoning of others.

Grade Span: 3 - 12

Purpose:
- Rich mathematical tasks for partner and group work encourage students to use the vocabulary to explain and discuss the task.
- Mathematical tasks that provide rich content and engage students allow them to not only understand what they have learned, but remember and apply it as well.

Description:
- Rich math tasks are open-ended and can be solved in many ways.
- Some math tasks are inquiry-based questions that have more than one correct answer or problems that require students to use hands-on materials to discover the solutions.
- 'Rich' or 'Low Threshold, High Ceiling' (LTHC) tasks that provide scope for students to move toward more sophisticated thinking skills.
    - Structured so that all students can make a start to the problem, even if needing support.
    - The 'low threshold' of such tasks reinforces understanding and fluency of a given concept and allows less confident learners to experience some level of success.

When:
- To ensure that students have the opportunity to engage in high-level thinking, teachers must regularly select and implement math tasks that promote reasoning and problem solving.

Why:
- Mathematical tasks promotes real understanding of important mathematical concepts.
- Facilitates the freeing of teacher time to be spent with less confident individuals if established collaborative problem solving guidelines such as:
    - time for quiet thinking and reflection,
    - 'ask three before you ask me',
    - pausing periodically to discuss the approaches of different students in various parts of the task (not asking for 'answers'!)
    - analysing errors as well as 'correct answers'

How:
To write your own rich tasks in maths in a grade and topic suited to your own classroom, the following development guidelines might prove useful.

1. Start with a closed version of a problem within a given topic or concept. Grade level textbooks and standardised tests are often a good source of closed mathematics problems.

2. 'Open up' the problem by removing or adapting parameters, to allow for a range of solutions.

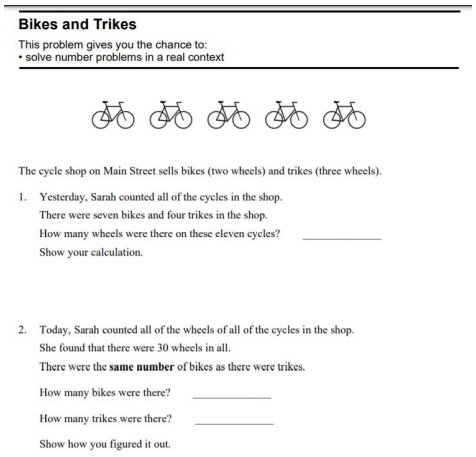
3. Push further by adding mathematical or procedural 'problem solving' complexity. The key concern here is to ensure a few different iterations of the same type of problem, each of which add increasingly complex levels of problem solving for students in the class.

Toolkit Strategy
Math Tasks (cont.)

4. Introduce a requirement for students to demonstrate reasoning and justification for a version or versions of the problem (have students challenge one another by setting their own versions of the tasks or have students compare several different methods of working out and write reasons for which they think one is 'better' than another; or have students design a model, experiment or product that applies the concept or topic in the real world).

5. Designing and using rich tasks in mathematics can be a rewarding and motivating experience for teachers as they watch their students engage in problem solving and demonstrate higher level thinking and can also provide a new and rich source of assessment information as teachers gain new insights into how their students are working mathematically.

Examples:



Bikes and Trikes

This problem gives you the chance to:
• solve number problems in a real context

The cycle shop on Main Street sells bikes (two wheels) and trikes (three wheels).

1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? _____
   Show your calculation.

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.
   How many bikes were there? _____
   How many trikes were there? _____
   Show how you figured it out.

MARS Tasks | Grade 4
https://www.scoe.org/files/mars-grade4.pdf

Additional Resources:

- Engaging Mathematical Tasks
- YouCubed Tasks
- A rubric describing rich mathematics tasks (the book Visible Learning for Mathematics by John Hattie, Douglas Fisher, and Nancy Frey):
- http://www.mrbartonmaths.com/teachers/rich-tasks/

Distance Learning Tools: Jamboard, Google Slides, Pear Deck, Nearpod, Padlet,Virtual Journal

“Graphic organizers support my ELLs because they provide a means of displaying complex concepts succinctly and visually.”

Elementary School Teacher

Toolkit Strategy:

Graphic Organizers

Toolkit Strategy
Graphic Organizers

SMP 5: Use appropriate tools strategically.
SMP 7: Look for and make use of structure.

Grade Span: T K - 12

Purpose:
- Use graphic organizers for math vocabulary to deepen understanding of key math vocabulary words in the math classroom.

Description:
- Graphic charts and tools used to visually represent and organize knowledge, key concepts, and key ideas.
- Depending on the focus of learning, the descriptors in the graphic organizers can be changed.
- They help students to visualize the relationships between words and their possible meanings

When:
- Use graphic organizers with explicit math vocabulary instruction.
- Use as assessments for learning because they give teachers a quick look at students' vocabulary knowledge.

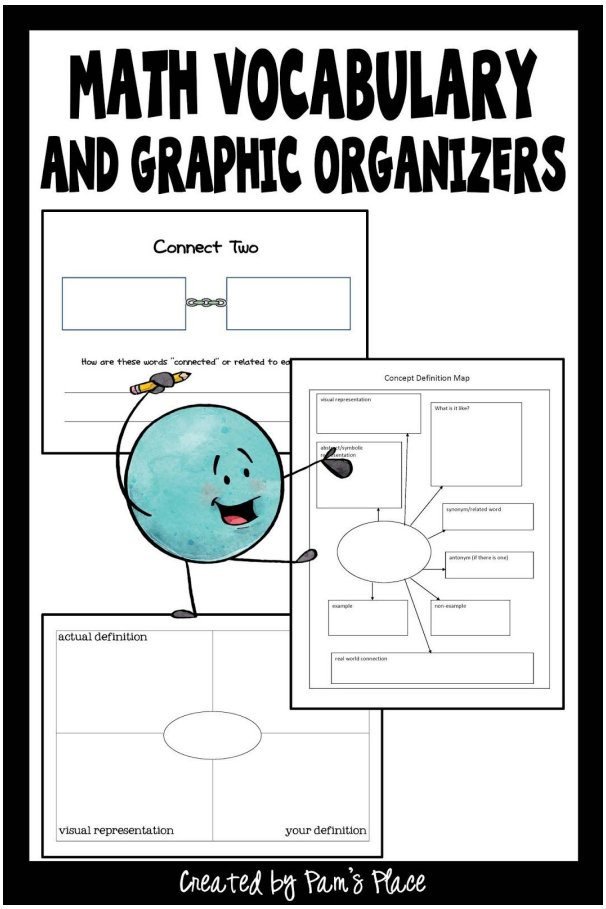
Why:
- Students learn mathematical vocabulary constructing meaning for mathematical vocabulary by actually doing authentic and meaningful mathematics.
- Teachers must be purposeful in constructing learning experiences that direct the student's attention to specific vocabulary.

How to use it:

- Teach students how to use the graphic organizer you've chosen for the activity. Explicitly explain the situations in which certain graphic organizers are used and why they are effective for use at those times.
- Use as a brainstorming tool to generate ideas before students begin an assignment, such as a math task, or a set of word problems to gather ideas or formalize procedures
- Give students time to practice using various graphic organizers. While one student may find T-charts especially helpful, another student might prefer Venn diagrams
- After exposing your students to several different types of graphic organizers and showing them the proper ways to use these organizers, you've given them a chance to find a new tool that they can use on their own for future projects.
- They can be an easy way to differentiate instruction for a variety of learning styles as well.

Toolkit Strategy
Graphic Organizers (cont.)

Examples:



Credit: Pam's Place

Additional Resources:

- Math Graphic Organizers Templates by Olena McLin

Distance Learning Tools: Jamboard, Google Slides, Pear Deck, Nearpod

“I LOVE math games! It’s a great change of routine, promotes student engagement and collaboration, encourages mathematical thinking, finding different strategies to solve and explore patterns in mathematical concepts. My students always look forward to playing math games!”

Middle School Teacher

Toolkit Strategy:

Math Games

Toolkit Strategy
Math Games

SMP 1: Make sense of problems and persevere in solving them.
SMP 8: Look for and express regularity in repeated reasoning.

Grade Span: K-12

Purpose:
- Engaging way for students to get the frequent practice required to build strong mental arithmetic skills and fact power.
- Used as a way to augment, not replace, activities that focus on rote practice of specific skills and facts.

Description:
- Give students opportunities to explore fundamental number concepts, such as the counting sequence, one-to-one correspondence, and computation strategies.
- Engaging mathematical games can also encourage students to explore number combinations, place value, patterns, and other important mathematical concepts.

When:
- Match a game to a specific mathematical objective
- Use games for specific purposes

Why:
- Provides meaningful situations for the application of mathematical skills.
- Students are motivated to participate and enjoy playing games.
- Games provide opportunities for building self-concept and developing positive attitudes towards mathematics through reducing the fear of failure and error.

How:
- Present a game (online or hands-on).
- Group students in pairs or triads to work on the task provided.
- Include explicit direction to go about the game.
- Encourage students to discuss steps/approaches/methods of doing the game.
- Include reflection questions to process the mathematical concepts presented in the game and connections made to learning new concepts.

Examples:
- Math Vocabulary Games: Math vocabulary games are designed to help students learn math terms and definitions.

Additional Resources:
- Math vocabulary spelling city
- Jeopardy
- Math Vocabulary Crossword Puzzle
- Math Flashcards

Distance Learning Tools: Kahoot! (Sample: Math vocabulary kahoot)

Section 6: Assessment for Learning

Section 6:

Assessment for Learning

Assessment is a critical step in the teaching and learning process for students, teachers, administrators, and families. The evolution of assessment continues to change the landscape of mathematics teaching and learning. Assessment for Learning (AFL) is a set of interrelated practices and processes that create conditions for learners to (1) develop their knowledge and skills in mathematics and other content areas, (2) foster greater student agency, and self-efficacy, and (3) fully engage in the learning spaces that validate and affirm their assets and identities.

Assessment for Learning shifts instructional practices intended to feature (1) clear learning goals and success criteria, so students understand what they are aiming for; (2) evidence of learning gathered during lesson to determine where students are relative to the goals; (3) pedagogical response to evidence, including descriptive feedback; (4) peer-and self-assessment to strengthen student's learning, efficacy, confidence and autonomy; and (5) a collaborative classroom culture where students and educators are partners in learning.

Beliefs About Mathematics Assessment

An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement. (NCTM Principles to Actions, p, 89)

NCTM defined ***assessment*** as the "process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes."

Below compares some unproductive and productive beliefs that influence assessment practices.

Productive Beliefs	Unproductive Beliefs
The primary purpose of assessment is to inform and improve the teaching and learning of mathematics.	The primary purpose of assessment is accountability for students through report card marks or grades.
Assessment is an ongoing process that is embedded in instruction to support student learning and make adjustments to instruction.	Assessment in the classroom is an interruption to the instructional process.
Mathematical understanding and processes can be measured through the use of a variety of assessment strategies and tasks.	Only multiple-choice and other "objective" paper-and-pencil tests can measure mathematical knowledge reliably and accurately.
Multiple data sources are needed to provide an accurate picture of teacher and student performance.	A single assessment can be used to make important decisions about students and teachers.
Assessment is a process that should help students become better judges of their own work, assist them in recognizing high-quality work when they produce it, and support them in using evidence to advance their own learning.	Assessment is something that is done to students.
Ongoing review and distributed practice within effective instruction are productive test preparation strategies.	Stopping teaching to review and take practice tests improves students' performance on high-stakes tests.

Source: NCTM Principles to Actions, pp. 91-92

Assessment for Learning

Two Types of Assessment

Formative assessment, commonly referred to as assessment *for* learning, has the goal of providing in-process information to teachers, and students, with regard to learning. Formative assessment is a process teachers and students use during instruction that provides feedback to adjust ongoing teaching moves and learning tactics. It is not a tool, an event, or a bank of test items or performance tasks.

The following definition of formative assessment comes from the *ELA/ELD Framework* (2014):

What is formative assessment? Formative assessment is a *process* teachers and students use *during* instruction that provides feedback to adjust ongoing teaching moves and learning tactics. It is *not* a tool or an event, nor a bank of test items or performance tasks. Well-supported by research evidence, it improves students' learning in time to achieve intended instructional outcomes. Key features include:

1. ***Clear lesson-learning goals and success criteria***, so students understand what they're aiming for;
2. ***Evidence of learning*** gathered *during lessons* to determine where students are relative to goals;
3. ***A pedagogical response to evidence, including descriptive feedback*** that supports learning by helping students answer: *Where am I going? Where am I now? What are my next steps?*
4. ***Peer- and self-assessment*** to strengthen students' learning, efficacy, confidence, and autonomy;
5. ***A collaborative classroom culture*** where students and teachers are partners in learning.

From Linquanti (2014, 2); Source: https://www.cde.ca.gov/ci/rl/cf/documents/elaeldfwchapter8.pdf

Summative assessment, commonly referred to as assessment *of* learning, has the goal of collecting information on a student's achievement *after* learning has occurred. Summative assessment measures include classroom, interim or benchmark assessments, and large-scale summative measures, such as the CAASPP or SAT.

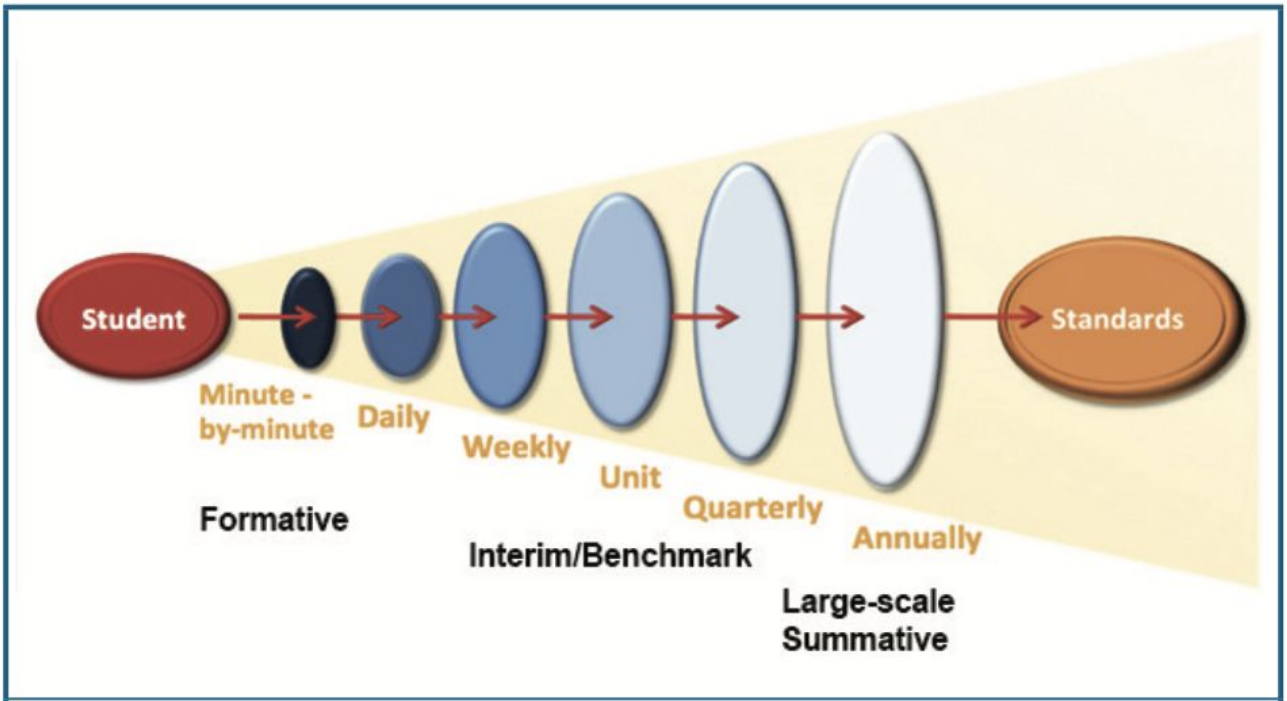
Summative assessments help determine whether students have attained a certain level of competency after a more or less extended period of instruction and learning; such as the end of a unit which may last several weeks, the end of a quarter, or annually (National Research Council [NRC] 2001).

Dimensions of Assessment for Learning

Dimension	Assessment *for* learning	Assessment *of* learning	Assessment *of* learning
Method	**Formative Assessment Process**	**Classroom Summative/ Interim/Benchmark Assessment***	**Large-scale Summative Assessment**
Main Purpose	Assist immediate learning (in the moment)	Measure student achievement or progress (may also inform future teaching and learning)	Evaluate educational programs and measure multi-year progress
Focus	Teaching and learning	Measurement	Accountability
Locus	Individual student and classroom learning	Grade level/ department/school	School/district/state
Priority for Instruction	High	Medium	Low
Proximity to learning	In-the-midst	Middle-distance	Distant
Timing	*During* immediate instruction or sequence of lessons	*After* teaching-learning cycle → *between* units/periodic	*End* of year/course
Participants	Teacher and Student (T-S / S-S / Self)	Student (may later include T-S in conference)	Student

Adapted from Linquanti (2014)

Purposes of Assessment Cycles



Source
Adapted from
Herman, Joan L., and Margaret Heritage. 2007. *Moving from Piecemeal to Effective Formative Assessment Practice: Moving Pictures on the Road to Student Learning.* Paper presented at the Council of Chief State School Officers Assessment Conference, Nashville, TN.

The Assessment Cycles by Purpose Graphic shows ways to assess overtime, including minute-by-minute, daily, weekly (each of which are formative assessment), weekly, unit, quarterly (each of which interim/benchmark), and annually (large-scale summative). It was adapted from work by Herman and Heritage (2007), and is copied from the 2014 *ELA/ELD Framework*.

Cycles of Assessment for Learning

Short Cycle	Methods	Information	Uses/Actions
Minute-by-minute	-Observation -Questions (teachers and students) -Instructional tasks -Student discussions -Written work/ representations	-Students' current learning status, relative difficulties and misunderstandings, emerging or partially formed ideas, full understanding	-Keep going, stop and find out more, provide oral feedback to individuals, adjust instructional moves in relation to student learning status (e.g., act on "teachable moments")
Daily Lesson	Planned and placed strategically in the lesson: -Observation -Questions (teachers and students) -Instructional tasks -Student discussions -Written work/ representations -Student self-reflection (e.g., quick write)	-Students' current learning status, relative difficulties and misunderstandings, emerging or partially formed ideas, full understanding	-Continue with planned instruction -Instructional adjustments in this or the next lesson -Find out more -Feedback to class or individual students (oral or written)
Week	-Student discussions and work products -Student self-reflection (e.g., journaling)	-Students' current learning status relative to lesson learning goals (e.g., have students met the goal(s), are they nearly there?	-Instructional planning for start of new week -Feedback to students (oral or written)

Cycles of Assessment for Learning

Medium Cycle	Methods	Information	Uses/Actions
End-of-Unit/ Project	-Student work artifacts (e.g., portfolio, writing project, oral presentation) -Use of rubrics -Student self-reflection (e.g., short survey) -Other classroom summative assessments designed by teacher(s)	-Status of student learning relative to unit learning goals-	-Grading -Reporting -Teacher reflection on effectiveness of planning and instruction -Teacher grade level/ departmental discussions of student work
Quarterly/ Interim/ Benchmark	-Portfolio -Oral reading observation -Test	-Status of achievement of intermediate goals toward meeting standards (results aggregated and disaggregated)	-Making within-year instructional decisions. -Monitoring, reporting; grading; same-year adjustments to curriculum programs -Teacher reflection on effectiveness of planning and instruction -Readjusting professional learning priorities and resource decisions

Cycles of Assessment for Learning

Long Cycle	Methods	Information	Uses/Actions
Annual	-Smarter Balanced Summative Assessment -CELDT -Portfolio -District/school created test	Status of student achievement with respect to standards (results aggregated and disaggregated)	-Judging students' overall learning -Gauging student, school, district, and state year-to-year progress -Monitoring, reporting and accountability - Classification and placement (e.g., ELs) -Certification -Adjustments to following year's instruction, curriculum, programs; -Final grades -Professional learning prioritization and resource decisions -Teacher reflection (individual/grade level/department) on overall effectiveness of planning and instruction

Mathematics Claims

Claim	Explanation
Claim 1	**Concepts and Procedures:** Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. This claim addresses procedural skills and the conceptual understanding on which the development of skills depends. It uses the cluster headings in the CA CCSSM as the targets of assessment for generating evidence for the claim. It is important to assess students' knowledge of how concepts are linked and why mathematical procedures work the way they do. Central to understanding this claim is making the connection to elements of these mathematical practices as stated in the CA CCSSM: SMP.5, SMP.6, SMP.7, and SMP.8.
Claim 2	**Problem Solving:** Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies. Assessment items and tasks focused on Claim 2 include problems in pure mathematics and problems set in context. Problems are presented as items and tasks that are well posed (that is, problem formulation is not necessary) and for which a solution path is not immediately obvious. These problems require students to construct their own solution pathway rather than follow a solution pathway that has been provided for them. Such problems are therefore unstructured, and students will need to select appropriate conceptual and physical tools to solve them.
Claim 3	**Communicating Reasoning:** Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others. Claim 3 refers to a recurring theme in the CA CCSSM content and practice standards: the ability to construct and present a clear, logical, and convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve justifications that are less formal. Assessment tasks that address this claim typically present a claim and ask students to provide a justification or counterexample.
Claim 4	**Modeling and Data Analysis:** Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems. Modeling is the bridge between "school math" and "the real world"—a bridge that has been missing from many mathematics curricula and assessments. Modeling is the twin of mathematical literacy, which is the focus of international comparison tests in mathematics given by the Programme for International Student Assessment (PISA). The CA CCSSM feature modeling as both a mathematical practice at all grade levels and a content focus in higher mathematics courses.

SBAC Math Claims Overview

Sample Performance Assessments - Grade 3

Grade 3

The number sentence below can be solved using tens and ones.

67 + 25 = __?__ tens and __?__ ones.

Select one number from each column to make the number sentence true.

Tens	Ones
○ 2	○ 2
○ 6	○ 5
○ 8	○ 10
○ 9	○ 12

Grade 3

For each expression in 1a – 1d, answer Yes or No if the expression is equivalent to the product of 7 and 9.

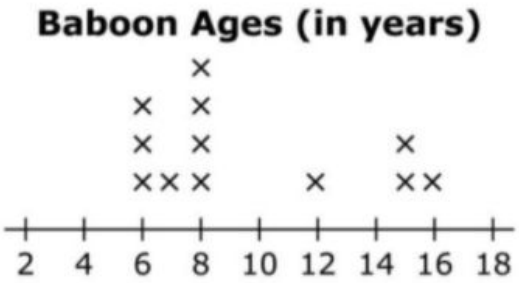
1a.	$7 \times (1 + 8)$	◯ Yes	◯ No
1b.	$9 \times (3 + 6)$	◯ Yes	◯ No
1c.	$(2 \times 5) + (5 \times 4)$	◯ Yes	◯ No
1d.	$(9 \times 2) + (9 \times 5)$	◯ Yes	◯ No

Sample 3rd-grade performance tasks

Sample Performance Assessments - Grade 4

Grade 4

A zookeeper made this line plot to show the ages of all the baboons at a zoo.



Part A

What fraction of the baboons at this zoo are eight years old?

$\frac{\square}{\square}$

Part B

What fraction of all the baboons at this zoo are **not** 8 years old?

$\frac{\square}{\square}$

[Sample 4th-grade performance tasks]

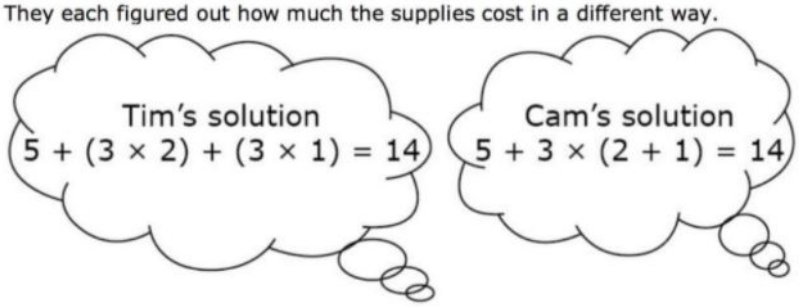
Sample Performance Assessments - Grades 4 & 5

Grade 4

Tim and Cam made posters for art class. They each used 1 poster board, 3 markers, and 3 feet of ribbon. The table shows the cost of their supplies.

Supplies	Cost
1 Poster Board	$5
3 Markers	$2 + $2 + $2
3 Feet of Ribbon	$1 + $1 + $1

They each figured out how much the supplies cost in a different way.



Which equation can be used to explain why Tim and Cam got the same result?

A. (3 x 2) + (3 x 1) = 3 x (2 + 1)
B. 5 + (3 x 2) = 3 x (2 + 1)
C. (3 x 2) = 3 x (2 + 1)
D. 3 x (5 + 1 + 1) = 5 + 3 x (2 + 1)

Grade 5

Pablo solved a multiplication problem using two different methods. He made a mistake in either Method W or Method Z.

Method W

23 × 49

20 × 9 = 180
3 × 9 = 27
20 × 4 = 80
3 × 4 = + 12
299

Method Z

23 × 49

Area Model

	40	+ 9
20	800	180
+ 3	120	27

Rectangle Sections

1
800
120
180
+ 27
1,127

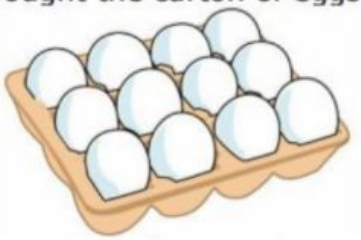
Identify the method where Pablo made a mistake and explain what he should do to correct it.

[Sample 5th-grade performance tasks]

Sample Performance Assessments - Grade 5

Grade 5

On Sunday, Travis bought the carton of eggs pictured below.



- On Monday, Travis used $\frac{1}{4}$ of the eggs in the carton.
- On Tuesday, Travis used $\frac{2}{3}$ of the eggs that **remained** in the carton after Monday.

Which picture represents the number of eggs remaining in the carton after Travis used eggs on Tuesday?

A

B

C

D

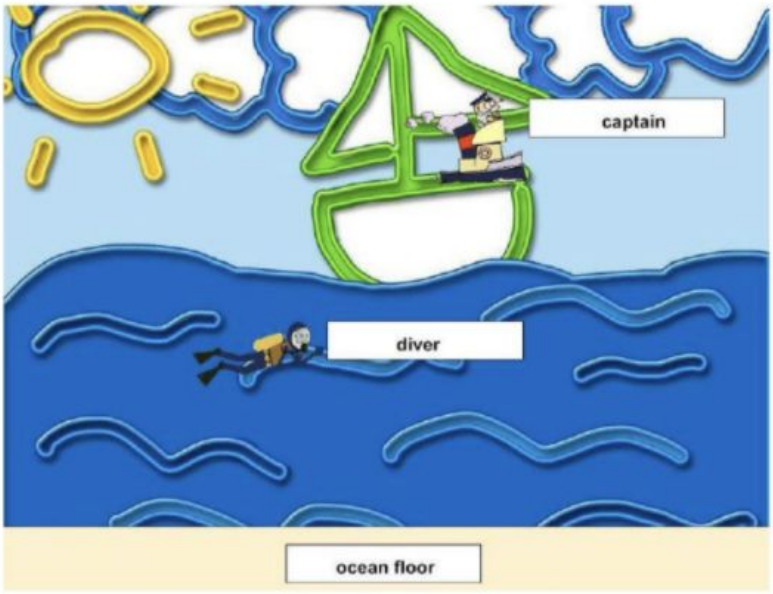
[Sample 5th-grade performance tasks]

Sample Performance Assessments - Grade 6

Grade 6

The level of the top of the water in the ocean is considered to be an altitude of zero (0) feet.

- The ocean floor at a particular dive site is – 20 feet.
- A diver is located at a – 5 feet at the same site.
- The captain of a boat is located at an altitude of 15 feet, directly above the diver.



Note: Illustration is not drawn to scale.

For numbers 1a – 1d, select True of False for each statement. Shade the circle that corresponds to your answer.

1a. The distance from the captain to the diver is greater than the distance from the top of the water to the ocean floor.

○ True ○ False

1b. The distance from the captain to the top of the water is the same as the distance from the diver to the ocean floor.

○ True ○ False

1c. When the diver swims to – 10 feet, the diver will be the same distance below the top of the water as the captain is above the top of the water.

○ True ○ False

1d. When the diver swims to – 10 feet, the diver's distance to the ocean floor will be equal to diver's distance to the top of the water.

○ True ○ False

[Sample 6th-grade performance tasks]

Sample Performance Assessments - Grade 7

Grade 7

Shelly incorrectly solves the equation $\frac{1}{2}(c + 6) = 7$. Her work is shown below.

A.

$\frac{1}{2}(c + 6) = 7$

Step 1: $\frac{1}{2}c + 6 = 7$

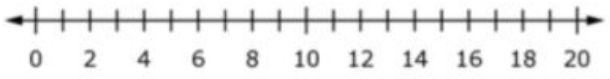
Step 2: $\frac{1}{2}c = 7 + 6$

Step 3: $\frac{1}{2}c = 13$

Step 4: $c = 13 \div 2$

Step 5: $c = 6\frac{1}{2}$

B. Correct solution



Part A

Select **all** the steps that show an error based on the equation in the previous step.

Answer: _____

Part B

Show the correct solution of the given equation (use the number line above).

Part C

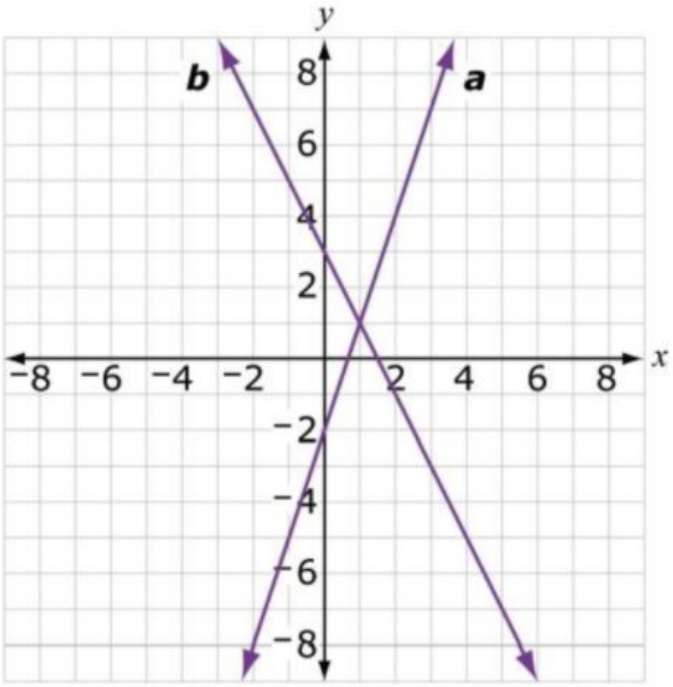
Show how you got your answer in Part B using the space below.

Sample 7th-grade performance tasks

Sample Performance Assessments Grade 8

Grade 8

The graphs of line ***a*** and line ***b*** are shown on this coordinate grid.



Match each line with each equation. Choose an equation and write it to the corresponding box for each line.

The equation of line a is _____

The equation of line b is _____

$y = -2x + 3$	$y = 2x + 3$	$y = -3x - 2$	$y = 3x - 2$
$y = \frac{1}{2}x + 3$	$y = -\frac{1}{2}x + 3$	$y = \frac{1}{3}x - 2$	$y = -\frac{1}{3}x - 2$

Sample Performance Assessments Grade 8

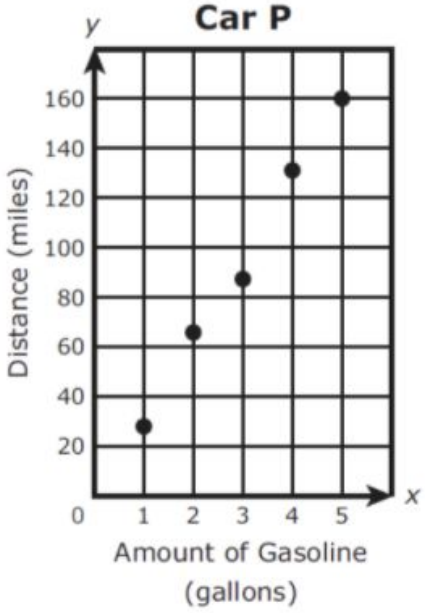
Grade 8

The gasoline mileage for two cars can be compared by finding the distance each car traveled and the amount of gasoline used. The table shows the distance that car M traveled using x gallons of gasoline. The graph shows the distance, y, that car P traveled using x gallons of gasoline.

Car M

Amount of Gasoline (gallons)	Distance (miles)
2	50.4
3	80.5
7	181.3
5	137.5

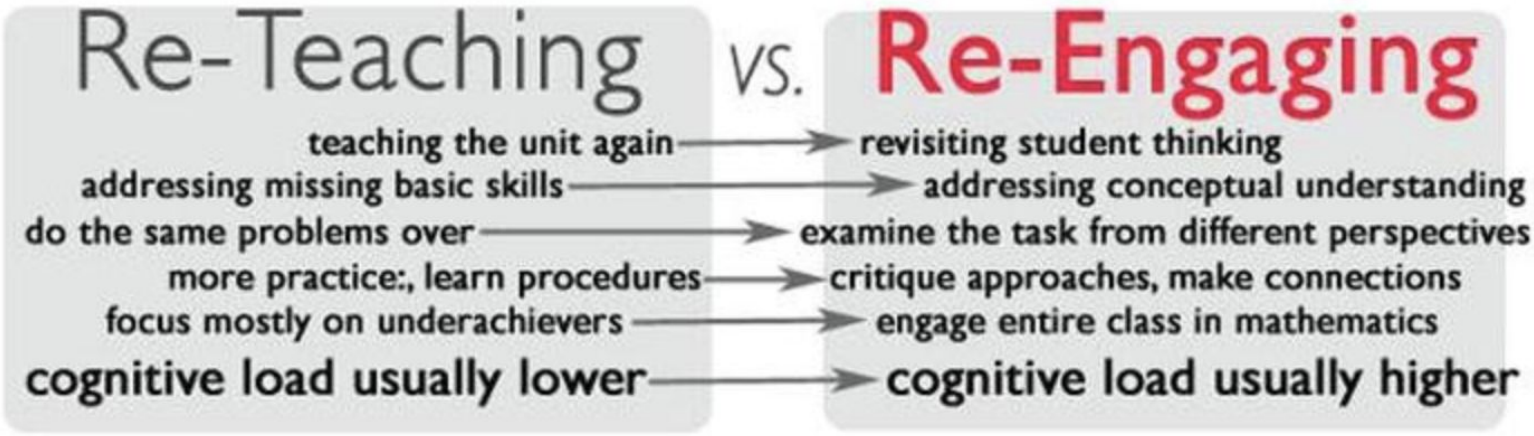
Car P



Based on the information in the table and the graph, compare the approximate miles per gallon of Car M to car P. Show your work and explain your answers using words.

Sample 8th-grade performance tasks

Re-engagement



Visual from Illustrative Mathematics

What is re-engagement?

Re-engagement is using student work for the purpose of uncovering current conceptions, providing feedback on student thinking, and helping students to go deeper into the mathematics. Students have the opportunity to reflect on their own learning while making connections between mathematical ideas. There is a focus on metacognitive development as students analyze other students' work in the search for possible different ways of conceptualizing the mathematics.

Why use re-engagement?

Re-engagement pushes students to address their conceptual understanding of a topic in order to make connections and clarify current conceptions. Research has shown it to be more effective than re-teaching the same content, because it engages all students in a metacognitive activity with high cognitive load.

When to use re-engagement?

Re-engagement should be used when students have had some opportunities to learn about a topic. At some point, students will have different conceptions of the topic. A re-engagement lesson allows all students to think again about the topic, deepening their understanding through a collaborative experience.

How do I re-engage students?

First, formatively assess students to identify common errors. Then, for each common error, ask, "What might students have been thinking?" By trying to understand the students' reasoning, a teacher can identify next steps. Opportunities for re-engagement range from small scale (10 minutes) to large scale (2 class periods).

Source: SFUSD Mathematics Department

References

References

Boaler, Jo. “Setting up Positive Norms in Math Class.” Youcubed, Stanford University, www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf.

CAMERON, SCHYRLET. Speaking and Listening Learning Stations, Grades 6- 8. MARK TWAIN MEDIA INC PUB, 2017.

Complex Instruction, complexinstruction.stanford.edu/.

“Culturally Responsive Teaching.” New America, www.newamerica.org/education-policy/reports/culturally-responsive-teaching/.

“EL Roadmap Policy and Printed Document.” English Learner Roadmap (CA Dept of Education), www.cde.ca.gov/sp/el/rm/rmpolicy.asp.

“English Learner Roadmap Toolkits.” Californians Together, 3 Dec. 2019, www.californianstogether.org/english-learner-roadmap-toolkits/.

Fostering Math Practices, www.fosteringmathpractices.com/.

“Integrating Technology into Your Classroom with the SAMR Model.” StarTime Studios, 8 Mar. 2018, startime.com.au/blog/2018/03/07/integrating-technology-classroom-samr-model/.

National Council of Teachers of Mathematics, 2014, pp. 91–92, Principles to Actions .

A Pathway to Equitable Mathematics Instruction, 27 Feb. 2021, equitablemath.org/.

“Remote Learning Resource - Setting Norms .” OpenSciEd.org, www.openscied.org/wp-content/uploads/2020/03/Resources-for-Remote-Teaching-Norms.pdf.

SFUSD Mathematics, 2021, www.sfusdmath.org/.

“Standards for Mathematical Practice.” Standards for Mathematical Practice | Common Core State Standards Initiative, 2021, www.corestandards.org/Math/Practice/.

Teaching for Robust Understanding Framework, truframework.org/.

“Technological, Pedagogical and Content Knowledge.” ICTEvangelist, 28 May 2013, ictevangelist.com/technological-pedagogical-and-content-knowledge/.

Zwiers, Jeff, et al. “Understanding Language/ Stanford Center for Assessment, Learning, and Equity.” Understanding Language/ SCALE, Stanford Graduate School of Education, 2017, ell.stanford.edu/sites/default/files/u6232/ULSCALE_ToA_Principles_MLRs__Final_v2.0_030217.pdf.